Chapter 4: Rational Expressions and Equations

Lesson 4.1: Equivalent Rational Expressions, page 222

1. a) i) $\frac{16x^3}{10x} = \frac{2x(8x^2)}{2x(5)}$

$\frac{16x^3}{10x} = \frac{8x^2}{5}, x \neq 0$

ii) $\frac{2x+6}{4} = \frac{2(x+3)}{2(2)}$

$\frac{2x+6}{4} = \frac{x+3}{2}$

iii) $\frac{x-2}{4x^2} = \frac{x-2}{4x^2} \cdot \frac{x}{x}$

$\frac{x-2}{4x^2} = \frac{x^2-2x}{4x^3}, x \neq 0$

iv) $\frac{x-3}{x-1} = \frac{x-3}{x-1} \cdot \frac{2}{2}$

$\frac{x-3}{x-1} = \frac{2x-6}{2x-2}, x \neq 1$

b) i) Denominator:

$10x = 0$

$x = 0$

The non-permissible value for this pair of equivalent expressions is $x = 0$.

ii) Denominator: $4 = 0$

This pair of expressions has no non-permissible values.

iii) Denominator:

$4x^2 = 0$

$x = 0$

The non-permissible value for this pair of equivalent expressions is $x = 0$.

iv) Denominator:

$x - 1 = 0$

$x = 1$

The non-permissible value for this pair of equivalent expressions is $x = 1$.

c) e.g., ii) substitute $x = 2$:

$\frac{2(2)+6}{4} = \frac{5}{2}, \frac{(2)+3}{2} = \frac{5}{2}$

Both expressions produce the same result for this value of x.

2. a) Let t represent the time of the journey, in hours.

$\text{Speed} = \frac{450}{t}$ km/h, $t > 0$

b) Let V represent the total volume melted, in cubic metres.

$\text{Volume} = \frac{V}{15}$ m^3/yr, $V \geq 0$

3. a) Denominator:

$-5 - x = 0$

$x = -5$

The non-permissible value for this pair of equivalent expressions is $x = -5$.

$\frac{2-7x}{-5-x} = \frac{2-7x}{-5-x} \cdot \frac{-1}{-1}$

$\frac{2-7x}{-5-x} = \frac{7x-2}{x+5}, x \neq -5$

$\frac{2-7x}{-5-x} = \frac{2-7x}{-5-x} \cdot \frac{2}{2}$

$\frac{2-7x}{-5-x} = \frac{4-14x}{-10-2x}, x \neq -5$

b) Denominator:

$a^2 + a = 0$

$a(a + 1) = 0$

$a = 0$ or $a + 1 = 0$

$a = 0$ or $a = -1$

The non-permissible values for this pair of equivalent expressions are $a = 0$ and $a = -1$.

$\frac{4a^2}{a^2+a} = \frac{a(4a)}{a(a+1)}$

$\frac{4a^2}{a^2+a} = \frac{4a}{a+1}, a \neq 0, -1$

$\frac{4a^2}{a^2+a} = \frac{4a^2}{a^2+a} \cdot \frac{3}{3}$

$\frac{4a^2}{a^2+a} = \frac{12a^2}{3a^2+3a}, a \neq 0, -1$

c) Denominator:

$3t^2 - t = 0$

$t(3t - 1) = 0$

$t = 0$ or $3t - 1 = 0$

$t = 0$ or $t = \frac{1}{3}$

The non-permissible values for this pair of equivalent expressions are $t = 0$ and $t = \frac{1}{3}$.

$\frac{4t^3}{3t^2-t} = \frac{t(4t^2)}{t(3t-1)}$

$\frac{4t^3}{3t^2-t} = \frac{4t^2}{3t-1}, t \neq 0, \frac{1}{3}$

$\frac{4t^3}{3t^2-t} = \frac{4t^3}{3t^2-t} \cdot \frac{-t}{-t}$

$\frac{4t^3}{3t^2-t} = \frac{-4t^4}{t^2-3t^3}, t \neq 0, \frac{1}{3}$

d) Denominator:
$4x - 8 = 0$
$x = 2$
The non-permissible value for this pair of equivalent expressions is $x = 2$.

$$\frac{-2x}{4x-8} = \frac{-2(x)}{-2(4-2x)}$$

$$\frac{-2x}{4x-8} = \frac{x}{4-2x}, x \neq 2$$

$$\frac{-2x}{4x-8} = \frac{-2x}{4x-8} \cdot \frac{1.5}{1.5}$$

$$\frac{-2x}{4x-8} = \frac{-3x}{6x-12}, x \neq 2$$

4. A. Non-permissible value:
$16a^2 = 0$
$a = 0$
D. Non-permissible value:
$64a^3 = 0$
$a = 0$
Both expressions have the same non-permissible value.

$$\text{Expression A} = \frac{4a-4}{16a^2}, a \neq 0$$

$$\text{Expression A} = \frac{4a-4}{16a^2} \cdot \frac{4a}{4a}$$

$$\text{Expression A} = \frac{16a^2-16a}{64a^3}, a \neq 0$$

Expression A = Expression D
Therefore, expressions A and D are equivalent.

B. Non-permissible value:
$32a^2 = 0$
$a = 0$
D. Non-permissible value:
$-128a^2 = 0$
$a = 0$
Both expressions have the same non-permissible value.

$$\text{Expression B} = \frac{2-2a}{32a^2}, a \neq 0$$

$$\text{Expression B} = \frac{2-2a}{32a^2} \cdot \frac{-4}{-4}$$

$$\text{Expression B} = \frac{8a-8}{-128a^2}, a \neq 0$$

Expression B = Expression C
Therefore, expressions B and C are equivalent.

5. The non-permissible value of the original expression is $x = \frac{1}{4}$.

A. Non-permissible value:
$4x - 1 = 0$
$4x = 1$
$x = \frac{1}{4}$
B. Non-permissible value:
$8x - 4 = 0$
$8x = 4$
$x = \frac{1}{2}$
C. Non-permissible value:
$x - 4 = 0$
$x = 4$
D. Non-permissible value:
$48x - 12 = 0$
$48x = 12$
$x = \frac{1}{4}$
Because expressions B and C have unique non-permissible values, they cannot be equivalent

$$\text{Expression A} = \frac{12x^2}{4x-1}, x \neq \frac{1}{4}$$

$$\text{Expression A} = \frac{12x^2}{4x-1} \cdot \frac{4}{4}$$

$$\text{Expression A} = \frac{48x^2}{16x-4}, x \neq \frac{1}{4}$$

Therefore, expression A is equivalent.
Expression D is not equivalent. To prove this, we will let $x = 1$ for both expressions.
When $x = 1$,

$$\frac{48x^2}{16x-4} = \frac{48(1)^2}{16(1)-4}$$

$$\frac{48x^2}{16x-4} = \frac{48}{16-4}$$

$$\frac{48x^2}{16x-4} = \frac{48}{12}$$

$$\frac{48x^2}{16x-4} = 4$$

When $x = 1$,

$$\frac{36x^2}{48x-12} = \frac{36(1)^2}{48(1)-12}$$

$$\frac{36x^2}{48x-12} = \frac{36}{48-12}$$

$$\frac{36x^2}{48x-12} = \frac{36}{36}$$

$$\frac{36x^2}{48x-12} = 1$$

Because these values are not equal, these expressions cannot be equivalent.

6. a) The fraction of the pool filled by the larger hose in one hour can be represented by $\frac{3}{3y-1}, y \neq \frac{1}{3}$.

b) y must be greater than $\frac{1}{3}$, because the rational expression must be positive.

7. Denominator:
$r^4 = 0$
$r = 0$
The non-permissible value for this expression is $r = 0$. However, in context, r must be greater than 0, because the radius of a blood vessel must be greater than 0.

8. Denominator:
$v - 5 = 0$
$v = 5$
The non-permissible value for this expression is $v = 5$. However, in context, v must be greater than 5, because this expression cannot be negative, because it represents time.

9. a) i) Denominator:
$x + 5 = 0$
$x = -5$
The non-permissible value for this expression is $x = -5$.
ii) Denominator:
$-3x - 15 = 0$
$-3x = 15$
$x = -5$
The non-permissible value for this expression is $x = -5$.
iii) Denominator:
$2x^3 + 10x^2 = 0$
$2x^2(x + 5) = 0$
$2x^2 = 0$ or $x + 5 = 0$
$x = 0$ or $x = -5$
The non-permissible values for this expression are $x = 0$ and $x = -5$.
b) These expressions are not equivalent, because expression iii) has an additional non-permissible value.

10. a) $\frac{7}{x}$

b) $\frac{2a+1}{3a-a^2}$

c) $\frac{2x^2+7}{3x^2+15x}$

d) $\frac{12}{3a-1}$

11. a) i) Denominator:
$3a - 2 = 0$
$3a = 2$
$a = \frac{2}{3}$
The non-permissible value for this expression is $a = \frac{2}{3}$.

ii) Denominator:
$16x^2 - 2x = 0$
$2x(8x - 1) = 0$
$2x = 0$ or $8x - 1 = 0$
$x = 0$ or $x = \frac{1}{8}$
The non-permissible values for this expression are $x = 0$ and $x = \frac{1}{8}$.

iii) Denominator:
$3y^3 = 0$
$y = 0$
The non-permissible value for this expression is $y = 0$.

b) i) $\frac{a+2}{3a-2} = \frac{a+2}{3a-2} \cdot \frac{-2}{-2}$

$\frac{a+2}{3a-2} = \frac{-4-2a}{4-6a}, a \neq \frac{2}{3}$

$\frac{a+2}{3a-2} = \frac{a+2}{3a-2} \cdot \frac{2}{2}$

$\frac{a+2}{3a-2} = \frac{2a+4}{6a-4}, a \neq \frac{2}{3}$

ii) $\frac{4x^4}{16x^2-2x} = \frac{2(2x^4)}{2(8x^2-x)}$

$\frac{4x^4}{16x^2-2x} = \frac{2x^4}{8x^2-x}, x \neq 0, \frac{1}{8}$

$\frac{4x^4}{16x^2-2x} = \frac{4x^4}{16x^2-2x} \cdot \frac{x}{x}$

$\frac{4x^4}{16x^2-2x} = \frac{4x^5}{16x^3-2x^2}, x \neq 0, \frac{1}{8}$

iii) $\frac{-15-6y}{3y^3} = \frac{-3(2y+5)}{-3(-y^3)} \cdot \frac{y}{y}$

$\frac{-15-6y}{3y^3} = \frac{2y^2+5y}{-y^4}, y \neq 0$

$\frac{-15-6y}{3y^3} = \frac{3(-5-2y)}{3(y^3)}$

$\frac{-15-6y}{3y^3} = \frac{-5-2y}{y^3}, y \neq 0$

12. e.g., To determine an equivalent rational expression, you need to multiply or divide the numerator and denominator by the same factor. This is similar to multiplying or dividing the numerator and denominator of a rational number by the same (numerical) factor.

$\frac{3x}{7-x}, x \neq 7$ is equivalent to $\frac{3x}{(7-x)} \cdot \frac{-2}{-2} = \frac{-6x}{2x-14}, x \neq 7$

$\frac{3}{5}$ is equivalent to $\frac{3}{5} \cdot \frac{3}{3} = \frac{9}{15}$

13. e.g., To determine an equivalent rational expression, you can multiply or divide the numerator and denominator of a rational expression by an algebraic factor, not just a number. The other difference is that the resulting rational expression is only equivalent if it has the same permissible values.

$\frac{2-4x}{8+6x}, x \neq \frac{-4}{3}$ is equivalent to

$\frac{2(1-2x)}{2(4+3x)} = \frac{1-2x}{4+3x}, x \neq \frac{-4}{3}$, but not to

$\frac{2-4x}{8+6x} \cdot \frac{x}{x} = \frac{2x-4x^2}{8x+6x^2}, x \neq \frac{-4}{3}, 0$.

14. $\frac{2x-7}{3x} = \frac{2x-7}{3x} \cdot \frac{x}{x}$

$\frac{2x-7}{3x} = \frac{2x^2-7x}{3x^2}, x \neq 0$

15. $\frac{4x^3-5x}{12x} = \frac{x(4x^2-5)}{x(12)}$

$\frac{4x^3-5x}{12x} = \frac{4x^2-5}{12}, x \neq 0$

16. a) Denominator:

$5a^2 - 7a = 0$

$a(5a - 7) = 0$

$a = 0$ or $5a - 7 = 0$

$a = 0$ or $a = \frac{7}{5}$

The non-permissible values for this expression are $a = 0$ and $a = \frac{7}{5}$.

b) Denominator:

$15x^2 + 10x = 0$

$5x(3x + 2) = 0$

$5x = 0$ or $3x + 2 = 0$

$x = 0$ or $x = -\frac{2}{3}$

The non-permissible values for this expression are $x = 0$ and $x = -\frac{2}{3}$.

c) Denominator:

$x^2 + 9 = 0$

$x^2 = -9$

$x = \pm\sqrt{-9}$

This situation is not possible, so this expression has no non-permissible values.

d) Denominator:

$4a^2 - 4 = 0$

$4a^2 = 4$

$a^2 = 1$

$a = \pm\sqrt{1}$

$a = 1$ or $a = -1$

The non-permissible values for this expression are $a = 1$ and $a = -1$.

17. a) $\frac{13}{2x-5x^2}, x \neq 0, \frac{2}{5}$

b) $\frac{3x}{12x-6x^2} = \frac{3(x)}{3(4x-2x^2)}, x \neq 0, 2$

$\frac{3x}{12x-6x^2} = \frac{x}{4x-2x^2}, x \neq 0, 2$

$\frac{3x}{12x-6x^2} = \frac{x(3)}{x(12-6x)}, x \neq 0, 2$

$\frac{3x}{12x-6x^2} = \frac{3}{12-6x}, x \neq 0, 2$

18. a) Let $r = 2$.

$2 + 2(2) + 2(2)^2 + 2(2)^3 + 2(2)^4 + 2(2)^5 = 126$

$\frac{2(1-(2)^6)}{1-2} = 126$

This rational expression is accurate for $r = 2$.

b) e.g., I expect that the rational expression will be $\frac{2(1-r^{10})}{1-r}, r \neq 1$. Let $r = 2$.

$2 + 2(2) + 2(2)^2 + 2(2)^3 + 2(2)^4 + 2(2)^5 + 2(2)^6 + 2(2)^7 + 2(2)^8 + 2(2)^9 = 2046$

$\frac{2(1-(2)^{10})}{1-2} = 2046$

This rational expression is accurate for $r = 2$.

Lesson 4.2: Simplifying Rational Expressions, page 229

1. A and C are simplified, because they cannot be reduced further. B can be simplified further:

$\frac{16x^2}{5x} = \frac{x(16x)}{x(5)}, x \neq 0$

$\frac{16x^2}{5x} = \frac{16x}{5}, x \neq 0$

D can also be simplified further:

$\frac{3a-6}{6-9a} = \frac{3(a-2)}{3(2-3a)}, a \neq \frac{2}{3}$

$\frac{3a-6}{6-9a} = \frac{a-2}{2-3a}, a \neq \frac{2}{3}$

2. a) Denominator:
$14x = 0$
$x = 0$
The non-permissible value is $x = 0$.

$$\frac{12x^4}{14x} = \frac{2x(6x^3)}{2x(7)},\ x \neq 0$$

$$\frac{12x^4}{14x} = \frac{6x^3}{7},\ x \neq 0$$

b) Denominator:
$8a^2(a + 3) = 0$
$8a^2 = 0$ or $a + 3 = 0$
$a = 0$ or $a = -3$
The non-permissible values are $a = 0$ and $a = -3$.

$$\frac{4a}{8a^2(a+3)} = \frac{4a}{4a(2a)(a+3)},\ a \neq -3, 0$$

$$\frac{4a}{8a^2(a+3)} = \frac{1}{2a(a+3)},\ a \neq -3, 0$$

c) Denominator:
$16(y - 2) = 0$
$y - 2 = 0$
$y = 2$
The non-permissible value is $y = 2$.

$$\frac{2y(2-y)}{16(y-2)} = \frac{-2y(y-2)}{16(y-2)},\ y \neq 2$$

$$\frac{2y(2-y)}{16(y-2)} = \frac{-y(2)(y-2)}{8(2)(y-2)}$$

$$\frac{2y(2-y)}{16(y-2)} = \frac{-y}{8},\ y \neq 2$$

d) Denominator:
$4x^3(3x + 4) = 0$
$4x^3 = 0$ or $3x + 4 = 0$

$$x = 0 \quad \text{or} \quad x = -\frac{4}{3}$$

The non-permissible values are $x = 0$ and $x = -\frac{4}{3}$.

$$\frac{(4x+3)4x^3}{4x^3(3x+4)} = \frac{4x^3(4x+3)}{4x^3(3x+4)},\ x \neq -\frac{4}{3}, 0$$

$$\frac{(4x+3)4x^3}{4x^3(3x+4)} = \frac{4x+3}{3x+4},\ x \neq -\frac{4}{3}, 0$$

3. a) $\dfrac{6y}{4y^3} = \dfrac{2y(3)}{2y(2y^2)},\ y \neq 0$

$$\frac{6y}{4y^3} = \frac{3}{2y^2},\ y \neq 0$$

b) $\dfrac{-17y^5}{7y^6} = \dfrac{y^5(-17)}{y^5(7y)},\ y \neq 0$

$$\frac{-17y^5}{7y^6} = \frac{-17}{7y},\ y \neq 0$$

c) $\dfrac{240y^2}{168y} = \dfrac{24y(10y)}{24y(7)},\ y \neq 0$

$$\frac{240y^2}{168y} = \frac{10y}{7},\ y \neq 0$$

d) $\dfrac{-32a^3}{26a^3} = \dfrac{2a^3(-16)}{2a^3(13)},\ a \neq 0$

$$\frac{-32a^3}{26a^3} = \frac{-16}{13},\ a \neq 0$$

4. a) $\dfrac{20a^2+30a^3}{25a^2} = \dfrac{5a^2(4+6a)}{5a^2(5)},\ a \neq 0$

$$\frac{20a^2+30a^3}{25a^2} = \frac{4+6a}{5},\ a \neq 0$$

b) $\dfrac{6a+2}{6a} = \dfrac{2(3a+1)}{2(3a)},\ a \neq 0$

$$\frac{6a+2}{6a} = \frac{3a+1}{3a},\ a \neq 0$$

$$\frac{6a+2}{6a} = \frac{3a}{3a} + \frac{1}{3a},\ a \neq 0$$

$$\frac{6a+2}{6a} = 1 + \frac{1}{3a},\ a \neq 0$$

c) $\dfrac{18a^2-24a^3}{36a^2} = \dfrac{6a^2(3-4a)}{6a^2(6)},\ a \neq 0$

$$\frac{18a^2-24a^3}{36a^2} = \frac{3-4a}{6},\ a \neq 0$$

d) $\dfrac{-15x^2-10x}{15x^2} = \dfrac{5x(-3x-2)}{5x(3x)},\ x \neq 0$

$$\frac{-15x^2-10x}{15x^2} = \frac{-3x-2}{3x},\ x \neq 0$$

$$\frac{-15x^2-10x}{15x^2} = \frac{-3x}{3x} - \frac{2}{3x},\ x \neq 0$$

$$\frac{-15x^2-10x}{15x^2} = -1 - \frac{2}{3x},\ x \neq 0$$

5. a) $\dfrac{4x^2-18x}{20x-90} = \dfrac{2x(2x-9)}{10(2x-9)},\ x \neq \dfrac{9}{2}$

$$\frac{4x^2-18x}{20x-90} = \frac{2x}{10},\ x \neq \frac{9}{2}$$

$$\frac{4x^2-18x}{20x-90} = \frac{x}{5},\ x \neq \frac{9}{2}$$

b) $\dfrac{9x^2-12x}{6x-8} = \dfrac{3x(3x-4)}{2(3x-4)},\ x \neq \dfrac{4}{3}$

$$\frac{9x^2-12x}{6x-8} = \frac{3x}{2},\ x \neq \frac{4}{3}$$

c) $\dfrac{10x^2-8x}{24x-30x^2} = \dfrac{2x(5x-4)}{2x(3)(4-5x)},\ x \neq 0,\ \dfrac{4}{5}$

$$\frac{10x^2-8x}{24x-30x^2} = \frac{5x-4}{3(4-5x)}$$

$$\frac{10x^2-8x}{24x-30x^2} = \frac{-(4-5x)}{3(4-5x)}$$

$$\frac{10x^2-8x}{24x-30x^2} = \frac{-1}{3},\ x \neq 0,\ \frac{4}{5}$$

d) $\frac{4x^2-8x}{10-5x}=\frac{4x(x-2)}{5(2-x)}, x\neq 2$

$\frac{4x^2-8x}{10-5x}=\frac{-4x(2-x)}{5(2-x)}$

$\frac{4x^2-8x}{10-5x}=\frac{-4x}{5}, x\neq 2$

6. a) $\frac{-10x^2-15x}{4x^3-9x.}=\frac{-5x(2x+3)}{x(4x^2-9)}, x\neq\frac{-3}{2},0,\frac{3}{2}$

$\frac{-10x^2-15x}{4x^3-9x}=\frac{-5x(2x+3)}{x(2x+3)(2x-3)}$

$\frac{-10x^2-15x}{4x^3-9x}=\frac{-5}{2x-3}, x\neq\frac{-3}{2}, 0,\frac{3}{2}$

b) e.g., Simplifying may remove a factor from the denominator that creates a non-permissible value.

7. a) Brad only divided one of the terms in the numerator by the denominator.

$\frac{4a+10}{2a}=\frac{4a}{2a}+\frac{10}{2a}, a\neq 0$

$\frac{4a+10}{2a}=2+\frac{5}{a}, a\neq 0$

b) e.g., You can factor one part of the numerator without factoring the other, as long as you split the fraction into two parts first. If you do not split the fraction, you will create the same type of error as Brad's.

8. a) Sharon did not begin her solution by determining the non-permissible values, so her solution is missing the non-permissible value of $x = 2$.

$\frac{12x-6x^2}{10x^3-20x^2}=\frac{3(2x)(2-x)}{5x(2x)(x-2)}, x\neq 0, 2$

$\frac{12x-6x^2}{10x^3-20x^2}=\frac{-3(2x)(x-2)}{5x(2x)(x-2)}$

$\frac{12x-6x^2}{10x^3-20x^2}=\frac{-3}{5x}, x\neq 0, 2$

b) Sharon's error may be a common error because you might forget to determine the non-permissible values before factoring and simplifying.

9. a) i) When $t = 0$

$\frac{4t}{4+2t}=\frac{4(0)}{4+2(0)}$

$\frac{4t}{4+2t}=0$

At 0 hours, the concentration of chlorine would be 0 mg/L.

ii) When $t = 2$

$\frac{4t}{4+2t}=\frac{4(2)}{4+2(2)}$

$\frac{4t}{4+2t}=\frac{8}{4+4}$

$\frac{4t}{4+2t}=\frac{8}{8}$

$\frac{4t}{4+2t}=1$

At 2 h, the concentration of chlorine would be 1 mg/L.

iii) When $t = 6$

$\frac{4t}{4+2t}=\frac{4(6)}{4+2(6)}$

$\frac{4t}{4+2t}=\frac{24}{4+12}$

$\frac{4t}{4+2t}=\frac{24}{16}$

$\frac{4t}{4+2t}=1.5$

At 6 h, the concentration of chlorine would be 1.5 mg/L.

b) $\frac{4t}{4+2t}=\frac{2(2t)}{2(2+t)}, t\neq -2$

$\frac{4t}{4+2t}=\frac{2t}{2+t}, t\neq -2$

c) Yes. e.g., Time cannot be negative.

10. a) $\frac{4A}{0.02A+0.24}$

b) When $A = 50$,

$\frac{4A}{0.02A+0.24}=\frac{4(50)}{0.02(50)+0.24}$

$\frac{4A}{0.02A+0.24}=\frac{200}{1+0.24}$

$\frac{4A}{0.02A+0.24}=\frac{200}{1.24}$

$\frac{4A}{0.02A+0.24}=161.290...$

Mike's profit per hour for a yard with an area of 50 m^2 is \$161.29.

c) $\frac{4A}{0.02A+0.24}=\frac{4A}{0.02A+0.24}\cdot\frac{50}{50}, A\neq -12$

$\frac{4A}{0.02A+0.24}=\frac{200A}{A+12}, A\neq -12$

d) Yes. e.g., There are no negative areas.

11. e.g., $\frac{3a-a^2}{2a-6}=\frac{a(3-a)}{2(a-3)}, a\neq 3$

$\frac{3a-a^2}{2a-6}=\frac{-a(a-3)}{2(a-3)}$

$\frac{3a-a^2}{2a-6}=\frac{-a}{2}, a\neq 3$

12. e.g., $\frac{x+1}{2x+2}=\frac{x+1}{2(x+1)}, x\neq -1$

$\frac{x+1}{2x+2}=\frac{1}{2}, x\neq -1$

13. a) $\frac{4x^3-36x}{6x^2-2x^3}=\frac{4x(x^2-9)}{2x^2(3-x)}, x\neq 0, 3$

$\frac{4x^3-36x}{6x^2-2x^3}=\frac{4x(x+3)(x-3)}{2x^2(3-x)}$

$\frac{4x^3-36x}{6x^2-2x^3}=\frac{2x(-2)(x+3)(3-x)}{2x(x)(3-x)}$

$\frac{4x^3-36x}{6x^2-2x^3}=\frac{-2(x+3)}{x}, x\neq 0, 3$

b) $\frac{121y^3-y}{y^2+11y^3}=\frac{y(121y^2-1)}{y^2(1+11y)}, y\neq \frac{-1}{11}, 0$

$\frac{121y^3-y}{y^2+11y^3}=\frac{y(11y+1)(11y-1)}{y^2(1+11y)}$

$\frac{121y^3-y}{y^2+11y^3}=\frac{y(11y+1)(11y-1)}{y(11y+1)(y)}$

$\frac{121y^3-y}{y^2+11y^3}=\frac{11y-1}{y}, y\neq \frac{-1}{11}, 0$

$\frac{121y^3-y}{y^2+11y^3}=\frac{11y}{y}-\frac{1}{y}, y\neq \frac{-1}{11}, 0$

$\frac{121y^3-y}{y^2+11y^3}=11-\frac{1}{y}, y\neq \frac{-1}{11}, 0$

14. e.g., To simplify a rational expression, you need to divide the numerator and denominator by all possible common factors. This is similar to dividing the numerator and denominator of a rational number by the largest possible common (numerical) factor.

$\frac{6x^2}{14x^2-2x}, x\neq 0, \frac{1}{7}$ simplifies to $\frac{3x}{7x-1}, x\neq 0, \frac{1}{7}$ after factoring $2x$ from the numerator and denominator.

15. a) The y-coordinate of vertex D is $x+5$, $x\neq -2$.
b) The only non-permissible value is -2; this corresponds to A and B both having coordinates $(0, 0)$, so that $ABCD$ is not a rectangle.

Lesson 4.3: Multiplying and Dividing Rational Expressions, page 238

1. a) $x-3=0$ or $3x-1=0$

$x=3$ or $x=\frac{1}{3}$

The non-permissible values are $x=3$ and $x=\frac{1}{3}$.

b) $x+4=0$ or $x-1=0$ or $x=0$

$x=-4$ or $x=1$ or $x=0$

The non-permissible values are $x=-4$, $x=0$ and $x=1$.

c) $7=0$ and $12=0$

There are no non-permissible values.

d) $x-1=0$ or $x=0$ or $x+3=0$

$x=1$ or $x=0$ or $x=-3$

The non-permissible values are $x=-3$, $x=0$ and $x=1$.

2. a) $\frac{4x^2}{27}\cdot\frac{9}{2x^3}=\frac{36x^2}{54x^3}, x\neq 0$

$\frac{4x^2}{27}\cdot\frac{9}{2x^3}=\frac{18x^2(2)}{18x^2(3x)}$

$\frac{4x^2}{27}\cdot\frac{9}{2x^3}=\frac{2}{3x}, x\neq 0$

b) $\left(\frac{5y^3}{12y}\right)\left(\frac{5}{15y^3}\right)=\left(\frac{y(5y^2)}{y(12)}\right)\left(\frac{5}{5(3y^3)}\right), y\neq 0$

$\left(\frac{5y^3}{12y}\right)\left(\frac{5}{15y^3}\right)=\left(\frac{5y^2}{12}\right)\left(\frac{1}{3y^3}\right)$

$\left(\frac{5y^3}{12y}\right)\left(\frac{5}{15y^3}\right)=\frac{5y^2}{36y^3}$

$\left(\frac{5y^3}{12y}\right)\left(\frac{5}{15y^3}\right)=\frac{5y^2}{(36y)y^2}$

$\left(\frac{5y^3}{12y}\right)\left(\frac{5}{15y^3}\right)=\frac{5}{36y}, y\neq 0$

c) $\frac{-6y^2}{18}\div\frac{3y^3}{14}=\frac{6(-y^2)}{6(3)}\div\frac{3y^3}{14}, y\neq 0$

$\frac{-6y^2}{18}\div\frac{3y^3}{14}=\frac{-y^2}{3}\div\frac{3y^3}{14}$

$\frac{-6y^2}{18}\div\frac{3y^3}{14}=\frac{-y^2}{3}\cdot\frac{14}{3y^3}$

$\frac{-6y^2}{18}\div\frac{3y^3}{14}=\frac{-14y^2}{9y^3}$

$\frac{-6y^2}{18}\div\frac{3y^3}{14}=\frac{-14y^2}{(9y)y^2}$

$\frac{-6y^2}{18}\div\frac{3y^3}{14}=\frac{-14}{9y}, y\neq 0$

d) $\frac{15x}{24}\div 8x^3=\frac{3(5x)}{3(8)}\div\frac{8x^3}{1}, x\neq 0$

$\frac{15x}{24}\div 8x^3=\frac{5x}{8}\div\frac{8x^3}{1}$

$\frac{15x}{24}\div 8x^3=\frac{5x}{8}\cdot\frac{1}{8x^3}$

$\frac{15x}{24}\div 8x^3=\frac{5x}{64x^3}$

$\frac{15x}{24}\div 8x^3=\frac{5x}{(64x^2)x}$

$\frac{15x}{24}\div 8x^3=\frac{5}{64x^2}, x\neq 0$

3. a) $\frac{-15y}{25y^4}\cdot 6y = \frac{5y(-3)}{5y(5y^3)}\cdot\frac{6y}{1},\ y\neq 0$

$\frac{-15y}{25y^4}\cdot 6y = \frac{-3}{5y^3}\cdot\frac{6y}{1}$

$\frac{-15y}{25y^4}\cdot 6y = \frac{-18y}{5y^3}$

$\frac{-15y}{25y^4}\cdot 6y = \frac{-18y}{(5y^2)y}$

$\frac{-15y}{25y^4}\cdot 6y = \frac{-18}{5y^2},\ y\neq 0$

b) $14a^2 \div \frac{4a^2}{7} = \frac{14a^2}{1}\div\frac{4a^2}{7},\ a\neq 0$

$14a^2 \div \frac{4a^2}{7} = \frac{14a^2}{1}\cdot\frac{7}{4a^2}$

$14a^2 \div \frac{4a^2}{7} = \frac{98a^2}{4a^2}$

$14a^2 \div \frac{4a^2}{7} = \frac{2a^2(49)}{2a^2(2)}$

$14a^2 \div \frac{4a^2}{7} = \frac{49}{2},\ a\neq 0$

c) $\frac{(-2x)^2}{6x^3}\div 12x = \frac{4x^2}{6x^3}\div\frac{12x}{1},\ x\neq 0$

$\frac{(-2x)^2}{6x^3}\div 12x = \frac{2x^2(2)}{2x^2(3x)}\div\frac{12x}{1}$

$\frac{(-2x)^2}{6x^3}\div 12x = \frac{2}{3x}\div\frac{12x}{1}$

$\frac{(-2x)^2}{6x^3}\div 12x = \frac{2}{3x}\cdot\frac{1}{12x}$

$\frac{(-2x)^2}{6x^3}\div 12x = \frac{2}{36x^2}$

$\frac{(-2x)^2}{6x^3}\div 12x = \frac{2}{2(18x^2)}$

$\frac{(-2x)^2}{6x^3}\div 12x = \frac{1}{18x^2},\ x\neq 0$

d) $\frac{24a}{18}\cdot\frac{1}{3a^3} = \frac{6(4a)}{6(3)}\cdot\frac{1}{3a^3},\ a\neq 0$

$\frac{24a}{18}\cdot\frac{1}{3a^3} = \frac{4a}{3}\cdot\frac{1}{3a^3}$

$\frac{24a}{18}\cdot\frac{1}{3a^3} = \frac{4a}{9a^3}$

$\frac{24a}{18}\cdot\frac{1}{3a^3} = \frac{4a}{(9a^2)a}$

$\frac{24a}{18}\cdot\frac{1}{3a^3} = \frac{4}{9a^2},\ a\neq 0$

4. a) $\left(\frac{27x^4}{14x}\right)\left(-\frac{21x^4}{6x^3}\right) = \left(\frac{(27x^3)x}{14x}\right)\left(-\frac{3x^3(7x)}{3x^3(2)}\right),\ x\neq 0$

$\left(\frac{27x^4}{14x}\right)\left(-\frac{21x^4}{6x^3}\right) = \left(\frac{27x^3}{14}\right)\left(\frac{-7x}{2}\right)$

$\left(\frac{27x^4}{14x}\right)\left(-\frac{21x^4}{6x^3}\right) = \frac{-189x^4}{28}$

$\left(\frac{27x^4}{14x}\right)\left(-\frac{21x^4}{6x^3}\right) = \frac{7(-27x^4)}{7(4)}$

$\left(\frac{27x^4}{14x}\right)\left(-\frac{21x^4}{6x^3}\right) = \frac{-27x^4}{4},\ x\neq 0$

b) $\frac{6a}{9}\div\frac{4a^2}{3} = \frac{3(2a)}{3(3)}\div\frac{4a^2}{3},\ a\neq 0$

$\frac{6a}{9}\div\frac{4a^2}{3} = \frac{2a}{3}\div\frac{4a^2}{3}$

$\frac{6a}{9}\div\frac{4a^2}{3} = \frac{2a}{3}\cdot\frac{3}{4a^2}$

$\frac{6a}{9}\div\frac{4a^2}{3} = \frac{6a}{12a^2}$

$\frac{6a}{9}\div\frac{4a^2}{3} = \frac{6a}{6a(2a)}$

$\frac{6a}{9}\div\frac{4a^2}{3} = \frac{1}{2a},\ a\neq 0$

c) $\frac{(3y^2)^2}{4y^3}\cdot\frac{20}{y} = \frac{9y^4}{4y^3}\cdot\frac{20}{y},\ y\neq 0$

$\frac{(3y^2)^2}{4y^3}\cdot\frac{20}{y} = \frac{(9y)y^3}{4y^3}\cdot\frac{20}{y}$

$\frac{(3y^2)^2}{4y^3}\cdot\frac{20}{y} = \frac{9y}{4}\cdot\frac{20}{y}$

$\frac{(3y^2)^2}{4y^3}\cdot\frac{20}{y} = \frac{180y}{4y}$

$\frac{(3y^2)^2}{4y^3}\cdot\frac{20}{y} = \frac{4y(45)}{4y}$

$\frac{(3y^2)^2}{4y^3}\cdot\frac{20}{y} = 45,\ y\neq 0$

d) $-\frac{15m}{20m^2}\div\frac{3}{14m} = -\frac{5m(3)}{5m(4m)}\div\frac{3}{14m},\ m\neq 0$

$-\frac{15m}{20m^2}\div\frac{3}{14m} = \frac{-3}{4m}\div\frac{3}{14m}$

$-\frac{15m}{20m^2}\div\frac{3}{14m} = \frac{-3}{4m}\cdot\frac{14m}{3}$

$-\frac{15m}{20m^2}\div\frac{3}{14m} = \frac{-42m}{12m}$

$-\frac{15m}{20m^2}\div\frac{3}{14m} = \frac{6m(-7)}{6m(2)}$

$-\frac{15m}{20m^2}\div\frac{3}{14m} = \frac{-7}{2},\ m\neq 0$

5. a) $\dfrac{3x(2x-1)}{7x^2(x-6)} \cdot \dfrac{14(6-x)}{8x^3}$

$= \dfrac{x(3)(2x-1)}{x(7x)(x-6)} \cdot \dfrac{2(7)(6-x)}{2(4x^3)}, x \neq 0, 6$

$= \dfrac{3(2x-1)}{7x(x-6)} \cdot \dfrac{7(6-x)}{4x^3}$

$= \dfrac{3(2x-1)}{7x(x-6)} \cdot \dfrac{7(-1)(x-6)}{4x^3}$

$= \dfrac{-21(2x-1)(x-6)}{28x^4(x-6)}$

$= \dfrac{7(x-6)(-3)(2x-1)}{7(x-6)(4x^4)}$

$= \dfrac{-3(2x-1)}{4x^4}, x \neq 0, 6$

b) $\dfrac{4b^2(2b+1)}{2b+3} \div \dfrac{10(1+2b)}{b+3}$

$= \dfrac{4b^2(2b+1)}{2b+3} \cdot \dfrac{b+3}{10(1+2b)}, b \neq -3, \dfrac{-3}{2}, \dfrac{-1}{2}$

$= \dfrac{4b^2(2b+1)(b+3)}{10(2b+3)(1+2b)}$

$= \dfrac{2(2b+1)(2b^2)(b+3)}{2(2b+1)(5)(2b+3)}$

$= \dfrac{2b^2(b+3)}{5(2b+3)}, b \neq -3, \dfrac{-3}{2}, \dfrac{-1}{2}$

6. a) $\dfrac{a^2+3a}{25} \cdot \dfrac{15a^3}{9a+3a^2}$

$= \dfrac{a(a+3)}{25} \cdot \dfrac{3a(5a^2)}{3a(3+a)}, a \neq -3, 0$

$= \dfrac{a(a+3)}{25} \cdot \dfrac{5a^2}{a+3}$

$= \dfrac{5a^3(a+3)}{25(a+3)}$

$= \dfrac{5a^3(a+3)}{5(5)(a+3)}$

$= \dfrac{a^3}{5}, a \neq -3, 0$

b) $\dfrac{y^4}{5y^2-2y^3} \cdot \dfrac{2y^3-5y^2}{y^3}$

$= \dfrac{y^2(y^2)}{y^2(5-2y)} \cdot \dfrac{y^2(2y-5)}{y^2(y)}, y \neq 0, \dfrac{5}{2}$

$= \dfrac{y^2}{5-2y} \cdot \dfrac{2y-5}{y}$

$= \dfrac{y^2(2y-5)}{y(5-2y)}$

$= \dfrac{y(-y)(5-2y)}{y(5-2y)}$

$= -y, y \neq 0, \dfrac{5}{2}$

c) $\dfrac{x+2x^2}{1-x} \cdot \dfrac{x^2-x}{2x^4+x^3}$

$= \dfrac{x(1+2x)}{1-x} \cdot \dfrac{x(x-1)}{x(x^2)(2x+1)}, x \neq \dfrac{-1}{2}, 0$

$= \dfrac{x(1+2x)}{1-x} \cdot \dfrac{x-1}{x^2(2x+1)}$

$= \dfrac{x(1+2x)(x-1)}{x^2(1-x)(2x+1)}$

$= \dfrac{x(2x+1)(-1)(1-x)}{x(x)(1-x)(2x+1)}$

$= \dfrac{-1}{x}, x \neq \dfrac{-1}{2}, 0$

$\dfrac{a^2-9}{3a+9} \cdot \dfrac{15a^3}{12a^2-36a}$

$= \dfrac{(a+3)(a-3)}{3(a+3)} \cdot \dfrac{3a(5a^2)}{3a(4)(a-3)}, a \neq -3, 0, 3$

d) $= \dfrac{a-3}{3} \cdot \dfrac{5a^2}{4(a-3)}$

$= \dfrac{5a^2(a-3)}{12(a-3)}$

$= \dfrac{5a^2}{12}, a \neq -3, 0, 3$

7. a) $\dfrac{6+x}{x-1} \div \dfrac{4x+24}{2x-2x^2}$

$= \dfrac{x+6}{x-1} \div \dfrac{2(2)(x+6)}{2x(1-x)}, x \neq -6, 0, 1$

$= \dfrac{x+6}{x-1} \div \dfrac{2(x+6)}{-x(x-1)}$

$= \dfrac{x+6}{x-1} \cdot \dfrac{-x(x-1)}{2(x+6)}$

$= \dfrac{-x(x+6)(x-1)}{2(x-1)(x+6)}$

$= \dfrac{-x}{2}, x \neq -6, 0, 1$

b) $\dfrac{4x^2+6x}{10x^3} \div \dfrac{2x+3}{15}$

$= \dfrac{2x(2x+3)}{2x(5x^2)} \div \dfrac{2x+3}{15}, x \neq \dfrac{-3}{2}, 0$

$= \dfrac{2x+3}{5x^2} \div \dfrac{2x+3}{15}$

$= \dfrac{2x+3}{5x^2} \cdot \dfrac{15}{2x+3}$

$= \dfrac{15(2x+3)}{5x^2(2x+3)}$

$= \dfrac{5(3)(2x+3)}{5(x^2)(2x+3)}$

$= \dfrac{3}{x^2}, x \neq \dfrac{-3}{2}, 0$

c) $\frac{25a^3}{10a^3-20a^2} \div \frac{-a}{4a-8}$

$= \frac{5a^2(5a)}{5a^2(2)(a-2)} \cdot \frac{4(a-2)}{-a}, a \neq 0, 2$

$= \frac{-20a(a-2)}{-2a(a-2)}$

$= -10, a \neq 0, 2$

d) $\frac{6x^4+x^3}{18x^2+6x} \div \frac{-4x^2-24x}{10x^3+30x^4}$

$= \frac{x(x^2)(6x+1)}{x(6)(3x+1)} \div \frac{2x(-2)(x+6)}{2x(5x^2)(1+3x)}, x \neq 0, \frac{-1}{3}, -6$

$= \frac{x^2(6x+1)}{6(3x+1)} \cdot \frac{-5x^2(3x+1)}{2(x+6)}$

$= \frac{-5x^4(6x+1)(3x+1)}{12(3x+1)(x+6)}$

$= \frac{-5x^4(6x+1)}{12(x+6)}, x \neq 0, \frac{-1}{3}, -6$

8. a) $\frac{-6x^2}{7x} \div \frac{21}{7x} = \frac{x(-6x)}{x(7)} \div \frac{7(3)}{7x}, x \neq 0$

$\frac{-6x^2}{7x} \div \frac{21}{7x} = \frac{-6x}{7} \cdot \frac{x}{3}$

$\frac{-6x^2}{7x} \div \frac{21}{7x} = \frac{-6x^2}{21}$

$\frac{-6x^2}{7x} \div \frac{21}{7x} = \frac{3(-2x^2)}{3(7)}$

$\frac{-6x^2}{7x} \div \frac{21}{7x} = \frac{-2x^2}{7}, x \neq 0$

b) $\frac{-2x^2}{7} \cdot \frac{9}{7x} = \frac{-18x^2}{49x}, x \neq 0$

$\frac{-2x^2}{7} \cdot \frac{9}{7x} = \frac{-18x}{49}, x \neq 0$

9. $\frac{5a^2}{10a^3-5a^2} \cdot \frac{3a-6}{a+2} = \frac{5a^2}{5a^2(2a-1)} \cdot \frac{3(a-2)}{a+2}, a \neq -2, 0, \frac{1}{2}$

$\frac{5a^2}{10a^3-5a^2} \cdot \frac{3a-6}{a+2} = \frac{1}{2a-1} \cdot \frac{3(a-2)}{a+2}$

$\frac{5a^2}{10a^3-5a^2} \cdot \frac{3a-6}{a+2} = \frac{3(a-2)}{(2a-1)(a+2)}, a \neq -2, 0, \frac{1}{2}$

10. e.g., Two expressions with this product are $\frac{4}{x-1}$ and $\frac{x-1}{x^2}$.

$\frac{4}{x-1} \cdot \frac{x-1}{x^2} = \frac{4(x-1)}{x^2(x-1)}, x \neq 0, 1$

$\frac{4}{x-1} \cdot \frac{x-1}{x^2} = \frac{4}{x^2}, x \neq 0, 1$

11. e.g., Two expressions with this quotient are $\frac{-12}{5x}$ and $\frac{2}{x^2}$.

$\frac{-12}{5x} \div \frac{2}{x^2} = \frac{-12}{5x} \cdot \frac{x^2}{2}, x \neq 0$

$\frac{-12}{5x} \div \frac{2}{x^2} = \frac{-12x^2}{10x}$

$\frac{-12}{5x} \div \frac{2}{x^2} = \frac{2x(-6x)}{2x(5)}$

$\frac{-12}{5x} \div \frac{2}{x^2} = \frac{-6x}{5}, x \neq 0$

12. e.g., Two expressions with this product are $\frac{2x-1}{x-2}$ and $\frac{3x}{2x-1}$

$\frac{2x-1}{x-2} \cdot \frac{3x}{2x-1} = \frac{3x(2x-1)}{(x-2)(2x-1)}, x \neq \frac{1}{2}, 2$

$\frac{2x-1}{x-2} \cdot \frac{3x}{2x-1} = \frac{3x}{x-2}, x \neq \frac{1}{2}, 2$

13. e.g., Two expressions with this quotient are $\frac{y-3}{3y-3y^2}$ and $\frac{3y}{1-y}$.

$\frac{y-3}{3y-3y^2} \div \frac{3y}{1-y} = \frac{y-3}{3y(1-y)} \cdot \frac{1-y}{3y}, y \neq 0, 1$

$\frac{y-3}{3y-3y^2} \div \frac{3y}{1-y} = \frac{(y-3)(1-y)}{9y^2(1-y)}$

$\frac{y-3}{3y-3y^2} \div \frac{3y}{1-y} = \frac{y-3}{9y^2}, y \neq 0, 1$

14. e.g., Two expressions with this product are $\frac{y+2}{4y}$ and $\frac{y^2}{3y^2+6y}$

$\frac{y+2}{4y} \cdot \frac{y^2}{3y^2+6y} = \frac{y+2}{4y} \cdot \frac{y(y)}{y(3)(y+2)}, y \neq -2, 0$

$\frac{y+2}{4y} \cdot \frac{y^2}{3y^2+6y} = \frac{y+2}{4y} \cdot \frac{y}{3(y+2)}$

$\frac{y+2}{4y} \cdot \frac{y^2}{3y^2+6y} = \frac{y(y+2)}{12y(y+2)}$

$\frac{y+2}{4y} \cdot \frac{y^2}{3y^2+6y} = \frac{1}{12}, y \neq -2, 0$

Either or both of the expressions are not defined at the non-permissible values, so their product cannot be defined at those values either.

15. a) $\dfrac{-6x}{15x^3} \div \dfrac{4(x-6)}{3x^3(6-x)}$

$= \dfrac{3x(-2)}{3x(5x^2)} \div \dfrac{4(x-6)}{-3x^3(x-6)}, x \neq 0, 6$

$= \dfrac{-2}{5x^2} \div \dfrac{4}{-3x^3}$

$= \dfrac{-2}{5x^2} \cdot \dfrac{-3x^3}{4}$

$= \dfrac{6x^3}{20x^2}$

$= \dfrac{2x^2(3x)}{2x^2(10)}$

$= \dfrac{3x}{10}, x \neq 0, 6$

b) $\dfrac{4a(a-1)(a-5)}{15a^3(5-a)} \cdot \dfrac{-1}{12a^2(a-1)}$

$= \dfrac{a(4)(a-1)(a-5)}{a(-15a^2)(a-5)} \cdot \dfrac{-1}{12a^2(a-1)}, a \neq 0, 1, 5$

$= \dfrac{-4(a-1)}{15a^2} \cdot \dfrac{-1}{12a^2(a-1)}$

$= \dfrac{4(a-1)}{180a^4(a-1)}$

$= \dfrac{4(a-1)}{4(45a^4)(a-1)}$

$= \dfrac{1}{45a^4}, a \neq 0, 1, 5$

Mid-Chapter Review, page 242

1. a) i) $\dfrac{6y}{9y^2} = \dfrac{y(6)}{y(9y)}$ $\dfrac{6y}{9y^2} = \dfrac{3(2y)}{3(3y^2)}$

$\dfrac{6y}{9y^2} = \dfrac{6}{9y}$ $\dfrac{6y}{9y^2} = \dfrac{2y}{3y^2}$

ii) $\dfrac{6}{6+x} = \dfrac{6}{6+x} \cdot \dfrac{6+x}{6+x}$ $\dfrac{6}{6+x} = \dfrac{6}{6+x} \cdot \dfrac{2}{2}$

$\dfrac{6}{6+x} = \dfrac{6(6+x)}{(6+x)^2}$ $\dfrac{6}{6+x} = \dfrac{12}{12+2x}$

iii) $\dfrac{-12x}{4x^2} = \dfrac{-12x}{4x^2} \cdot \dfrac{x}{x}$ $\dfrac{-12x}{4x^2} = \dfrac{4x(-3)}{4x(x)}$

$\dfrac{-12x}{4x^2} = \dfrac{-12x^2}{4x^3}$ $\dfrac{-12x}{4x^2} = \dfrac{-3}{x}$

iv) $\dfrac{x+3}{2x-1} = \dfrac{x+3}{2x-1} \cdot \dfrac{2}{2}$ $\dfrac{x+3}{2x-1} = \dfrac{x+3}{2x-1} \cdot \dfrac{-1}{-1}$

$\dfrac{x+3}{2x-1} = \dfrac{2x+6}{4x-2}$ $\dfrac{x+3}{2x-1} = \dfrac{-x-3}{1-2x}$

b) i) Denominator:

$9y^2 = 0$

$y = 0$

The non-permissible value is $y = 0$.

ii) Denominator:

$6 + x = 0$

$x = -6$

The non-permissible value is $x = -6$.

iii) Denominator:

$4x^2 = 0$

$x = 0$

The non-permissible value is $x = 0$.

iv) Denominator:

$2x - 1 = 0$

$2x = 1$

$x = \dfrac{1}{2}$

The non-permissible value is $x = \dfrac{1}{2}$.

2. First expression's denominator:

$12x + 9 = 0$

$12x = 9$

$x = \dfrac{3}{4}$

The non-permissible value of the first expression is $x = \dfrac{3}{4}$.

Second expression's denominator:

$8x^2 + 6x = 0$

$2x(4x + 3) = 0$

$2x = 0$ or $4x + 3 = 0$

$x = 0$ or $x = \dfrac{3}{4}$

The non-permissible values of the second expression are $x = 0$ and $x = \dfrac{3}{4}$.

Third expression's denominator:

$4x + 3 = 0$

$4x = 3$

$x = \dfrac{3}{4}$

The non-permissible value of the third expression is $x = \dfrac{3}{4}$.

The second expression has an extra non-permissible value, so it cannot be equivalent to either of the other expressions.

First expression $= \dfrac{3x-6}{12x+9}, x \neq \dfrac{3}{4}$

First expression $= \dfrac{3(x-2)}{3(4x+3)}$

First expression $= \dfrac{x-2}{4x+3}, x \neq \dfrac{3}{4}$

First expression = Third expression

The first and third expressions are equivalent. The first expression is 3 times the second expression.

3. a) Let t represent the time it took Joanna to ride the route on her old bike, in hours. Her speed can be represented with the expression $\frac{85}{t-\frac{1}{3}}$ km/h.

b) $\frac{85}{t-\frac{1}{3}}=\frac{85}{t-\frac{1}{3}}\cdot\frac{3}{3}$

$\frac{85}{t-\frac{1}{3}}=\frac{255}{3t-1}$ km/h

c) The non-permissible value is $t=\frac{1}{3}$ h, e.g., when $t=\frac{1}{3}$ h, the expression is not defined.

d) $t\neq\frac{1}{3}$

4. a) Denominator: $10a = 0$, $a = 0$. The non-permissible value is $a = 0$.

$\frac{2a^2-32a}{10a}=\frac{2a(a-16)}{2a(5)}, a\neq 0$

$\frac{2a^2-32a}{10a}=\frac{a-16}{5}, a\neq 0$

b) Denominator: $-6b = 0$, $b = 0$. The non-permissible value is $b = 0$.

$\frac{-4-6b}{-6b}=\frac{-2(2+3b)}{-2(3b)}, b\neq 0$

$\frac{-4-6b}{-6b}=\frac{2+3b}{3b}, b\neq 0$

c) Denominator: $15y = 0$, $y = 0$. The non-permissible value is $y = 0$.

$\frac{15y+5}{15y}=\frac{5(3y+1)}{5(3y)}, y\neq 0$

$\frac{15y+5}{15y}=\frac{3y+1}{3y}, y\neq 0$

d) Denominator: $2y = 0$, $y = 0$. The non-permissible value is $y = 0$.

$\frac{3y^3-9y}{2y}=\frac{y(3y^2-9)}{y(2)}, y\neq 0$

$\frac{3y^3-9y}{2y}=\frac{3y^2-9}{2}, y\neq 0$

5. a) $\frac{x^2-25}{4x+20}=\frac{(x+5)(x-5)}{4(x+5)}, x\neq -5$

$\frac{x^2-25}{4x+20}=\frac{x-5}{4}, x\neq -5$

b) $\frac{3x^2-15x}{6x-30}=\frac{3x(x-5)}{6(x-5)}, x\neq 5$

$\frac{3x^2-15x}{6x-30}=\frac{x}{2}, x\neq 5$

c) $\frac{x-1}{3-3x}=\frac{x-1}{3(1-x)}, x\neq 1$

$\frac{x-1}{3-3x}=\frac{-(1-x)}{3(1-x)}$

$\frac{x-1}{3-3x}=\frac{-1}{3}, x\neq 1$

d) $\frac{3a^2-a^3}{1-3a}=\frac{a^2(3-a)}{1-3a}, a\neq\frac{1}{3}$

6. a) $\frac{165000-750A}{3A+3450}=\frac{3(55\,000-250A)}{3(A+1150)}$

$\frac{165000-750A}{3A+3450}=\frac{55\,000-250A}{A+1150}$

b) Let $A = 17$

$VO_2\ \max=\frac{55\,000-250A}{A+1150}$

$VO_2\ \max=\frac{55\,000-250(17)}{17+1150}$

$VO_2\ \max=\frac{50\,750}{1167}$

$VO_2\ \max = 43.487...$ mL/min/kg

The estimated VO_2 max for a person my age is around 43.5 mL/min/kg.

c) Yes. e.g., Age cannot be negative.

7. a) $\frac{15x^3}{8x}\cdot\frac{6x}{9x^3}=\frac{x(15x^2)}{x(8)}\cdot\frac{3x(2)}{3x(3x^2)}, x\neq 0$

$\frac{15x^3}{8x}\cdot\frac{6x}{9x^3}=\frac{15x^2}{8}\cdot\frac{2}{3x^2}$

$\frac{15x^3}{8x}\cdot\frac{6x}{9x^3}=\frac{30x^2}{24x^2}$

$\frac{15x^3}{8x}\cdot\frac{6x}{9x^3}=\frac{6x^2(5)}{6x^2(4)}$

$\frac{15x^3}{8x}\cdot\frac{6x}{9x^3}=\frac{5}{4}, x\neq 0$

b) $5a^3\div\frac{10a^2}{3}=\frac{5a^3}{1}\cdot\frac{3}{10a^2}, a\neq 0$

$5a^3\div\frac{10a^2}{3}=\frac{15a^3}{10a^2}$

$5a^3\div\frac{10a^2}{3}=\frac{5a^2(3a)}{5a^2(2)}$

$5a^3\div\frac{10a^2}{3}=\frac{3a}{2}, a\neq 0$

8. a) $\dfrac{3x(3x-1)}{21(x-6)} \cdot \dfrac{14(6-x)}{2(3x-1)}$

$= \dfrac{x(3x-1)}{7(x-6)} \cdot \dfrac{7(6-x)}{(3x-1)}, x \neq \dfrac{1}{3}, 6$

$= \dfrac{x(3x-1)}{7(x-6)} \cdot \dfrac{-7(x-6)}{(3x-1)}$

$= \dfrac{-7x(3x-1)(x-6)}{7(x-6)(3x-1)}$

$= \dfrac{7(-x)(3x-1)(x-6)}{7(3x-1)(x-6)}$

$= -x, x \neq \dfrac{1}{3}, 6$

b) $\dfrac{b+1}{2b+3} \div \dfrac{10(1+b)}{b+3} = \dfrac{b+1}{2b+3} \cdot \dfrac{b+3}{10(b+1)}, b \neq -3, \dfrac{-3}{2}, -1$

$\dfrac{b+1}{2b+3} \div \dfrac{10(1+b)}{b+3} = \dfrac{(b+1)(b+3)}{10(b+1)(2b+3)}$

$\dfrac{b+1}{2b+3} \div \dfrac{10(1+b)}{b+3} = \dfrac{b+3}{10(2b+3)}, b \neq -3, \dfrac{-3}{2}, -1$

9. a) $\dfrac{x-3}{4} \div \dfrac{1}{4x-12} = \dfrac{x-3}{4} \div \dfrac{1}{4(x-3)}, x \neq 3$

$\dfrac{x-3}{4} \div \dfrac{1}{4x-12} = \dfrac{x-3}{4} \cdot \dfrac{4(x-3)}{1}$

$\dfrac{x-3}{4} \div \dfrac{1}{4x-12} = \dfrac{4(x-3)^2}{4}$

$\dfrac{x-3}{4} \div \dfrac{1}{4x-12} = (x-3)^2, x \neq 3$

b) $\dfrac{2y^2+6y}{8y+2} \cdot \dfrac{16y^2-1}{12y^2}$

$= \dfrac{2y(y+3)}{2(4y+1)} \cdot \dfrac{(4y+1)(4y-1)}{12y^2}, y \neq \dfrac{-1}{4}, 0$

$= \dfrac{y(y+3)}{(4y+1)} \cdot \dfrac{(4y+1)(4y-1)}{12y^2}$

$= \dfrac{y(y+3)(4y+1)(4y-1)}{12y^2(4y+1)}$

$= \dfrac{y(4y+1)(y+3)(4y-1)}{y(4y+1)(12y)}$

$= \dfrac{(y+3)(4y-1)}{12y}, y \neq \dfrac{-1}{4}, 0$

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1. a) $10x^2$ **b)** $18x^3$ **c)** $x(x-1)$ **d)** $(x+1)(x-1)(2x-1)$

2. a) $5x^2 = 0$ or $10x = 0$

$x = 0$ or $x = 0$

The non-permissible value for this pair of equivalent expressions is $x = 0$.

b) $6x^3 = 0$ or $9x = 0$

$x = 0$ or $x = 0$

The non-permissible value for this pair of equivalent expressions is $x = 0$.

c) $x(x-1) = 0$ or $x - 1 = 0$

$x = 0$ or $x - 1 = 0$

$x = 0$ or $x = 1$

The non-permissible values for this pair of equivalent expressions are $x = 0$ and $x = 1$.

d) $(x+1)(2x-1) = 0$ or $(2x-1)(x-1) = 0$

$x + 1 = 0$ or $2x - 1 = 0$ or $x - 1 = 0$

$x = -1$ or $x = \dfrac{1}{2}$ or $x = 1$

The non-permissible values for this pair of equivalent expressions are $x = -1$, $x = \dfrac{1}{2}$ and $x = 1$.

3. a) $\dfrac{12x^2}{5x} - \dfrac{6x^2}{5x} = \dfrac{12x}{5} - \dfrac{6x}{5}, x \neq 0$

$\dfrac{12x^2}{5x} - \dfrac{6x^2}{5x} = \dfrac{12x-6x}{5}$

$\dfrac{12x^2}{5x} - \dfrac{6x^2}{5x} = \dfrac{6x}{5}, x \neq 0$

b) $\dfrac{5x+1}{x-1} + \dfrac{3x-1}{x-1} = \dfrac{5x+1+3x-1}{x-1}, x \neq 1$

$\dfrac{5x+1}{x-1} + \dfrac{3x-1}{x-1} = \dfrac{8x}{x-1}, x \neq 1$

c) $\dfrac{1}{x^2+2x} - \dfrac{5x+3}{x^2+2x} = \dfrac{1-5x-3}{x^2+2x}, x \neq 0, -2$

$\dfrac{1}{x^2+2x} - \dfrac{5x+3}{x^2+2x} = \dfrac{-5x-2}{x(x+2)}, x \neq 0, -2$

d) $\dfrac{3y+7}{y(y-2)} + \dfrac{4-8y}{(y-2)y} = \dfrac{3y+7}{y(y-2)} + \dfrac{4-8y}{y(y-2)}, y \neq 2, 0$

$\dfrac{3y+7}{y(y-2)} + \dfrac{4-8y}{(y-2)y} = \dfrac{3y+7+4-8y}{y(y-2)}$

$\dfrac{3y+7}{y(y-2)} + \dfrac{4-8y}{(y-2)y} = \dfrac{-5y+11}{y(y-2)}$

$\dfrac{3y+7}{y(y-2)} + \dfrac{4-8y}{(y-2)y} = \dfrac{11-5y}{y(y-2)}, y \neq 2, 0$

4. a) $\dfrac{-6a+5}{4a^2} + \dfrac{3}{6a} = \dfrac{-6a+5}{4a^2} + \dfrac{1}{2a}, a \neq 0$

$\dfrac{-6a+5}{4a^2} + \dfrac{3}{6a} = \dfrac{-6a+5}{4a^2} + \left(\dfrac{1}{2a} \cdot \dfrac{2a}{2a}\right)$

$\dfrac{-6a+5}{4a^2} + \dfrac{3}{6a} = \dfrac{-6a+5}{4a^2} + \dfrac{2a}{4a^2}$

$\dfrac{-6a+5}{4a^2} + \dfrac{3}{6a} = \dfrac{-6a+5+2a}{4a^2}$

$\dfrac{-6a+5}{4a^2} + \dfrac{3}{6a} = \dfrac{5-4a}{4a^2}, a \neq 0$

b) $\frac{3y-7}{6y}-\frac{1}{12y^2}=\left(\frac{3y-7}{6y}\cdot\frac{2y}{2y}\right)-\frac{1}{12y^2}$, $y\neq 0$

$\frac{3y-7}{6y}-\frac{1}{12y^2}=\frac{6y^2-14y}{12y^2}-\frac{1}{12y^2}$

$\frac{3y-7}{6y}-\frac{1}{12y^2}=\frac{6y^2-14y-1}{12y^2}$, $y\neq 0$

c) $\frac{4}{3x}+5=\frac{4}{3x}+\left(5\cdot\frac{3x}{3x}\right)$, $x\neq 0$

$\frac{4}{3x}+5=\frac{4}{3x}+\frac{15x}{3x}$

$\frac{4}{3x}+5=\frac{15x+4}{3x}$, $x\neq 0$

d) $\frac{3b}{12}-\frac{5}{4b}=\frac{b}{4}-\frac{5}{4b}$, $b\neq 0$

$\frac{3b}{12}-\frac{5}{4b}=\left(\frac{b}{4}\cdot\frac{b}{b}\right)-\frac{5}{4b}$

$\frac{3b}{12}-\frac{5}{4b}=\frac{b^2}{4b}-\frac{5}{4b}$

$\frac{3b}{12}-\frac{5}{4b}=\frac{b^2-5}{4b}$, $b\neq 0$

5. a) $\frac{5x+12}{x(x+6)}-\frac{3}{x}=\frac{5x+12}{x(x+6)}-\left(\frac{3}{x}\cdot\frac{x+6}{x+6}\right)$, $x\neq -6,0$

$\frac{5x+12}{x(x+6)}-\frac{3}{x}=\frac{5x+12}{x(x+6)}-\frac{3x+18}{x(x+6)}$

$\frac{5x+12}{x(x+6)}-\frac{3}{x}=\frac{5x+12-3x-18}{x(x+6)}$

$\frac{5x+12}{x(x+6)}-\frac{3}{x}=\frac{2x-6}{x(x+6)}$

$\frac{5x+12}{x(x+6)}-\frac{3}{x}=\frac{2(x-3)}{x(x+6)}$

b) $\frac{-2}{2y-3}+\frac{1}{4(y+1)}$

$=\left(\frac{-2}{2y-3}\cdot\frac{4(y+1)}{4(y+1)}\right)+\left(\frac{1}{4(y+1)}\cdot\frac{2y-3}{2y-3}\right)$, $y\neq -1,\frac{3}{2}$

$=\frac{-8y-8}{4(2y-3)(y+1)}+\frac{2y-3}{4(2y-3)(y+1)}$

$=\frac{-8y-8+2y-3}{4(2y-3)(y+1)}$

$\frac{-6y-11}{4(2y-3)(y+1)}$, $y\neq -1,\frac{3}{2}$

c) $\frac{4y-6}{(y-6)(y+6)}-\frac{12-3y}{6(y-6)}$

$=\frac{4y-6}{(y-6)(y+6)}-\frac{4-y}{2(y-6)}$, $y\neq -6, 6$

$=\left(\frac{4y-6}{(y-6)(y+6)}\cdot\frac{2}{2}\right)-\left(\frac{4-y}{2(y-6)}\cdot\frac{y+6}{y+6}\right)$

$=\frac{8y-12}{2(y-6)(y+6)}-\frac{-y^2-2y+24}{2(y-6)(y+6)}$

$=\frac{8y-12+y^2+2y-24}{2(y-6)(y+6)}$

$=\frac{y^2+10y-36}{2(y-6)(y+6)}$, $y\neq -6, 6$

d) $\frac{-2}{a+2}+\frac{5}{a-2}$

$=\left(\frac{-2}{a+2}\cdot\frac{a-2}{a-2}\right)+\left(\frac{5}{a-2}\cdot\frac{a+2}{a+2}\right)$, $a\neq -2, 2$

$=\frac{-2a+4}{(a+2)(a-2)}+\frac{5a+10}{(a+2)(a-2)}$

$=\frac{-2a+4+5a+10}{(a+2)(a-2)}$

$=\frac{3a+14}{(a+2)(a-2)}$, $a\neq -2, 2$

6. a) $\frac{6}{x^2-1}+\frac{1}{x+1}=\frac{6}{(x+1)(x-1)}+\frac{1}{x+1}$, $x\neq -1, 1$

$\frac{6}{x^2-1}+\frac{1}{x+1}=\frac{6}{(x+1)(x-1)}+\left(\frac{1}{x+1}\cdot\frac{x-1}{x-1}\right)$

$\frac{6}{x^2-1}+\frac{1}{x+1}=\frac{6}{(x+1)(x-1)}+\frac{x-1}{(x+1)(x-1)}$

$\frac{6}{x^2-1}+\frac{1}{x+1}=\frac{6+x-1}{(x+1)(x-1)}$

$\frac{6}{x^2-1}+\frac{1}{x+1}=\frac{x+5}{(x+1)(x-1)}$, $x\neq -1, 1$

b) $\frac{3a}{4a+2}-\frac{3a^2-6}{4a^2-1}$

$=\frac{3a}{2(2a+1)}-\frac{3a^2-6}{(2a+1)(2a-1)}$, $a\neq \frac{-1}{2},\frac{1}{2}$

$=\left(\frac{3a}{2(2a+1)}\cdot\frac{2a-1}{2a-1}\right)-\left(\frac{3a^2-6}{(2a+1)(2a-1)}\cdot\frac{2}{2}\right)$

$=\frac{6a^2-3a}{2(2a+1)(2a-1)}-\frac{6a^2-12}{2(2a+1)(2a-1)}$

$=\frac{6a^2-3a-6a^2+12}{2(2a+1)(2a-1)}$

$=\frac{3(4-a)}{2(2a+1)(2a-1)}$, $a\neq \frac{-1}{2},\frac{1}{2}$

c) $\frac{9y}{3y+18}+\frac{y-1}{y+6}=\frac{3(3y)}{3(y+6)}+\frac{y-1}{y+6}, y\neq -6$

$\frac{9y}{3y+18}+\frac{y-1}{y+6}=\frac{3y}{y+6}+\frac{y-1}{y+6}$

$\frac{9y}{3y+18}+\frac{y-1}{y+6}=\frac{3y+y-1}{y+6}$

$\frac{9y}{3y+18}+\frac{y-1}{y+6}=\frac{4y-1}{y+6}, y\neq -6$

d) $\frac{a}{2a^2-2}-\frac{a}{2a^2}=\frac{a}{2(a^2-1)}-\frac{1}{2a}, a\neq -1, 0, 1$

$\frac{a}{2a^2-2}-\frac{a}{2a^2}=\left(\frac{a}{2(a^2-1)}\cdot\frac{a}{a}\right)-\left(\frac{1}{2a}\cdot\frac{a^2-1}{a^2-1}\right)$

$\frac{a}{2a^2-2}-\frac{a}{2a^2}=\frac{a^2}{2a(a^2-1)}-\frac{a^2-1}{2a(a^2-1)}$

$\frac{a}{2a^2-2}-\frac{a}{2a^2}=\frac{a^2-a^2+1}{2a(a^2-1)}$

$\frac{a}{2a^2-2}-\frac{a}{2a^2}=\frac{1}{2a(a+1)(a-1)}, a\neq -1, 0, 1$

7. a) $\frac{7}{2x-6}+\frac{4}{10x-15}=\frac{7}{2(x-3)}+\frac{4}{5(2x-3)}, x\neq\frac{3}{2}, 3$

$\frac{7}{2x-6}+\frac{4}{10x-15}=\left(\frac{7}{2(x-3)}\cdot\frac{5(2x-3)}{5(2x-3)}\right)+\left(\frac{4}{5(2x+3)}\cdot\frac{2(x-3)}{2(x-3)}\right)$

$\frac{7}{2x-6}+\frac{4}{10x-15}=\frac{70x+105}{10(x-3)(2x-3)}+\frac{8x-24}{10(x-3)(2x-3)}$

$\frac{7}{2x-6}+\frac{4}{10x-15}=\frac{70x-105+8x-24}{10(x-3)(2x-3)}$

$\frac{7}{2x-6}+\frac{4}{10x-15}=\frac{78x-129}{10(x-3)(2x-3)}, x\neq\frac{3}{2}, 3$

b) $\frac{5}{2b-2}-\frac{4+b}{2b^2-2b}=\frac{5}{2(b-1)}-\frac{4+b}{2b(b-1)}, b\neq 0, 1$

$\frac{5}{2b-2}-\frac{4+b}{2b^2-2b}=\left(\frac{5}{2(b-1)}\cdot\frac{b}{b}\right)-\frac{4+b}{2b(b-1)}$

$\frac{5}{2b-2}-\frac{4+b}{2b^2-2b}=\frac{5b}{2b(b-1)}-\frac{4+b}{2b(b-1)}$

$\frac{5}{2b-2}-\frac{4+b}{2b^2-2b}=\frac{5b-b-4}{2b(b-1)}$

$\frac{5}{2b-2}-\frac{4+b}{2b^2-2b}=\frac{4b-4}{2b(b-1)}$

$\frac{5}{2b-2}-\frac{4+b}{2b^2-2b}=\frac{4(b-1)}{2b(b-1)}$

$\frac{5}{2b-2}-\frac{4+b}{2b^2-2b}=\frac{2}{b}, b\neq 0, 1$

8. a) $\frac{3a}{2a+1}-\frac{3}{4a^2-1}=\frac{3a}{2a+1}-\frac{3}{(2a+1)(2a-1)}, a\neq\frac{-1}{2}, \frac{1}{2}$

$\frac{3a}{2a+1}-\frac{3}{4a^2-1}=\left(\frac{3a}{2a+1}\cdot\frac{2a-1}{2a-1}\right)-\frac{3}{(2a+1)(2a-1)}$

$\frac{3a}{2a+1}-\frac{3}{4a^2-1}=\frac{6a^2-3a}{(2a+1)(2a-1)}-\frac{3}{(2a+1)(2a-1)}$

$\frac{3a}{2a+1}-\frac{3}{4a^2-1}=\frac{6a^2-3a-3}{(2a+1)(2a-1)}$

$\frac{3a}{2a+1}-\frac{3}{4a^2-1}=\frac{3(2a^2-a-1)}{(2a+1)(2a-1)}$

$\frac{3a}{2a+1}-\frac{3}{4a^2-1}=\frac{3(2a+1)(a-1)}{(2a+1)(2a-1)}$

$\frac{3a}{2a+1}-\frac{3}{4a^2-1}=\frac{3(a-1)}{(2a-1)}, a\neq\frac{-1}{2}, \frac{1}{2}$

b) $\frac{x^2+9}{9-x^2}-\frac{x}{3-x}$

$=\frac{x^2+9}{(3+x)(3-x)}-\frac{x}{3-x}, x\neq -3, 3$

$=\frac{x^2+9}{(3+x)(3-x)}-\left(\frac{x}{3-x}\cdot\frac{3+x}{3+x}\right)$

$=\frac{x^2+9}{(3+x)(3-x)}-\frac{3x+x^2}{(3+x)(3-x)}$

$=\frac{x^2+9-3x-x^2}{(3+x)(3-x)}$

$=\frac{9-3x}{(3+x)(3-x)}$

$=\frac{3(3-x)}{(3+x)(3-x)}$

$=\frac{3}{(3+x)}, x\neq -3, 3$

9. a) Marlene did not put each expression on the LCD before performing the subtraction. She also forgot to distribute the negative sign within the binomial of the second expression.

b) $\frac{2x}{2x+1}-\frac{2x-1}{x+3}$

$=\left(\frac{2x}{2x+1}\cdot\frac{x+3}{x+3}\right)-\left(\frac{2x-1}{x+3}\cdot\frac{2x+1}{2x+1}\right), x\neq -3, \frac{-1}{2}$

$=\frac{2x^2+6x}{(2x+1)(x+3)}-\frac{4x^2-1}{(2x+1)(x+3)}$

$=\frac{2x^2+6x-4x^2+1}{(2x+1)(x+3)}$

$=\frac{-2x^2+6x+1}{(2x+1)(x+3)}, x\neq -3, \frac{-1}{2}$

10. Let v represent Charlie's paddling speed, in kilometres per hour. The total time of his trip can be represented by $\frac{4}{v-7}+\frac{4}{v+7}$, $v>7$ or $\frac{8v}{(v+7)(v-7)}$, $v>7$.

11. Let p represent the stock price on Tuesday, in dollars. The difference in the number of shares bought can be represented by $\frac{3000}{p-0.15}-\frac{3000}{p}$, $p>0.15$ or $\frac{450}{p(p-0.15)}$.

12. e.g., First, make sure that the expressions are in simplest terms. Then, determine any common factors in the denominators and the unique factors in each denominator. The LCD is the product of all these factors.

$$\frac{3}{x+1}+\frac{2}{x+2}=\left(\frac{3}{x+1}\cdot\frac{x+2}{x+2}\right)+\left(\frac{2}{x+2}\cdot\frac{x+1}{x+1}\right),\ x\neq -2,-1$$
$$\frac{3}{x+1}+\frac{2}{x+2}=\frac{3x+6}{(x+1)(x+2)}+\frac{2x+2}{(x+1)(x+2)}$$
$$\frac{3}{x+1}+\frac{2}{x+2}=\frac{3x+6+2x+2}{(x+1)(x+2)}$$
$$\frac{3}{x+1}+\frac{2}{x+2}=\frac{5x+8}{(x+1)(x+2)}$$

13.
$$\left(\frac{x^2-x-6}{2x-3}\right)\left(\frac{3}{x-3}-\frac{9x}{x^2-9}\right)$$
$$=\left(\frac{(x+2)(x-3)}{2x-3}\right)\left(\frac{3}{x-3}-\frac{9x}{(x+3)(x-3)}\right),\ x\neq -3,\frac{3}{2},3$$
$$=\left(\frac{(x+2)(x-3)}{2x-3}\right)\left[\left(\frac{3}{x-3}\right)\cdot\left(\frac{x+3}{x+3}\right)-\frac{9x}{(x+3)(x-3)}\right]$$
$$=\left(\frac{(x+2)(x-3)}{2x-3}\right)\left(\frac{3x+9}{(x+3)(x-3)}-\frac{9x}{(x+3)(x-3)}\right)$$
$$=\left(\frac{(x+2)(x-3)}{2x-3}\right)\left(\frac{3x+9-9x}{(x+3)(x-3)}\right)$$
$$=\left(\frac{(x+2)(x-3)}{2x-3}\right)\left(\frac{-3(2x-3)}{(x+3)(x-3)}\right)$$
$$=\frac{-3(x+2)(x-3)(2x-3)}{(2x-3)(x-3)(x+3)}$$
$$=\frac{-3(x+2)}{x+3},\ x\neq -3,\frac{3}{2},3$$

14.
$$\left(\frac{1}{a^2}-\frac{1}{9}\right)\div\frac{1}{3a}=\left[\left(\frac{1}{a^2}\cdot\frac{9}{9}\right)-\left(\frac{1}{9}\cdot\frac{a^2}{a^2}\right)\right]\div\frac{1}{3a},\ a\neq 0$$
$$\left(\frac{1}{a^2}-\frac{1}{9}\right)\div\frac{1}{3a}=\left(\frac{9}{9a^2}-\frac{a^2}{9a^2}\right)\div\frac{1}{3a}$$
$$\left(\frac{1}{a^2}-\frac{1}{9}\right)\div\frac{1}{3a}=\frac{9-a^2}{9a^2}\div\frac{1}{3a}$$
$$\left(\frac{1}{a^2}-\frac{1}{9}\right)\div\frac{1}{3a}=\frac{(3+a)(3-a)}{(3a)^2}\cdot\frac{3a}{1}$$
$$\left(\frac{1}{a^2}-\frac{1}{9}\right)\div\frac{1}{3a}=\frac{3a(3+a)(3-a)}{(3a)^2}$$
$$\left(\frac{1}{a^2}-\frac{1}{9}\right)\div\frac{1}{3a}=\frac{(3+a)(3-a)}{3a},\ a\neq 0$$

Applying Problem-Solving Strategies, page 251

Sample Answers:

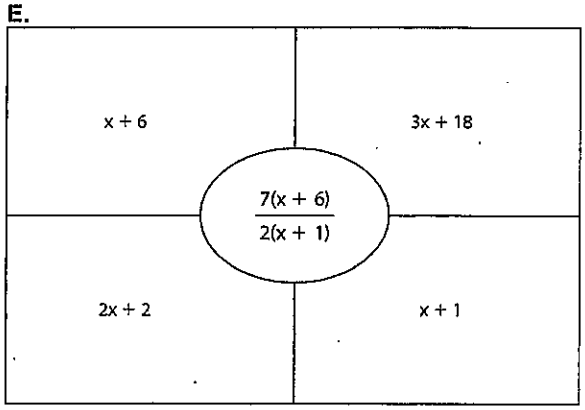
A. $\frac{x^2-x}{x-1}$ and $\frac{5}{x+4}$

B.
$$\frac{x^2-x}{x-1}+\frac{5}{x+4}=\frac{x^2+4x+5}{x+4}$$
$$\frac{x^2-x}{x-1}-\frac{5}{x+4}=\frac{x^2+4x-5}{x+4}$$
$$\frac{x^2-x}{x-1}\cdot\frac{5}{x+4}=\frac{5x}{x+4}$$
$$\frac{x^2-x}{x-1}\div\frac{5}{x+4}=\frac{x^2+4x}{5}$$

C. None of these expressions result in $\frac{4}{x}$, so I need to create new rational expressions. I will try $\frac{5}{x-1}$ and $\frac{x+4}{x^2-x}$.
$$\frac{5}{x-1}+\frac{x+4}{x^2-x}=\frac{6x+4}{x(x-1)}$$
$$\frac{5}{x-1}-\frac{x+4}{x^2-x}=\frac{4}{x}$$

D. Once I did the problem with a random selection of expressions, I noticed that two expressions shared a common factor, $(x-1)$. I decided to try two rational expressions with this factor as part of the denominator. The resulting expression would be easy to add or subtract. The denominators of the expressions also had the common factor of x, which is in the solution $\frac{4}{x}$.

E.



History Connection, page 252

A. Students may choose to hold the lens at arm's length until the image of what they are looking at is in focus. Then they can measure the distance between their eye and the lens, d_i. Alternatively, students could place the lens between a candle and a screen (a piece of paper) and adjust the position of the screen until the image of the candle is inverted and in focus on the screen. Then they could measure the distance, d_i, between the screen and the lens.

B. For $d_o = 100$ cm and $d_i = 20$ cm:

$$\frac{1}{f} = \frac{1}{100} + \frac{1}{20}$$

$$\frac{1}{f} \cdot 100f = \left(\frac{1}{100} + \frac{1}{20}\right) \cdot 100f$$

$$100 = f + 5f$$

$$100 = 6f$$

$$\frac{50}{3} = f$$

C. For $d_o = 100$ cm:

$$f = \frac{100d_i}{d_i + 100},\ d_i \neq -100, 0$$

D. Verify the new formula for $d_o = 100$ cm and $d_i = 20$ cm:

$$f = \frac{100(20)}{(20) + 100}$$

$$f = \frac{50}{3}$$

E. The non-permissible values are $d_o = 0, -d_i$ and $d_i = 0, -d_o$. The non-permissible values prevent division by zero. Values of d_o and d_i smaller than zero are inadmissible (distance cannot be negative).

Lesson 4.5: Solving Rational Equations, page 258

1. a) The non-permissible value is $x = 0$.

$$\frac{1}{x} + \frac{1}{3x} = \frac{1}{6},\ x \neq 0$$

$$\frac{1}{x}(3x) + \frac{1}{3x}(3x) = \frac{1}{6}(3x)$$

$$3 + 1 = \frac{3x}{6}$$

$$4 = \frac{x}{2}$$

$$4(2) = \frac{x}{2}(2)$$

$$8 = x$$

The solution to this equation is $x = 8$.

b) The non-permissible value is $y = 2$.

$$\frac{2y-6}{4y-8} = 1,\ y \neq 2$$

$$\frac{2y-6}{4y-8}(4y-8) = 1(4y-8)$$

$$2y - 6 = 4y - 8$$

$$2y - 4y = 6 - 8$$

$$-2y = -2$$

$$\frac{-2y}{-2} = \frac{-2}{-2}$$

$$y = 1$$

The solution is $y = 1$.

c) The non-permissible values are $x = 0$ and $x = -1$.

$$\frac{1}{x} - \frac{1}{x+1} = \frac{1}{6},\ x \neq 0, -1$$

$$\frac{1}{x}(x)(x+1) - \frac{1}{x+1}(x)(x+1) = \frac{1}{6}(x)(x+1)$$

$$x + 1 - x = \frac{1}{6}(x^2 + x)$$

$$(1)(6) = \frac{1}{6}(x^2 + x)(6)$$

$$6 = x^2 + x$$

$$0 = x^2 + x - 6$$

$$0 = (x+3)(x-2)$$

$$x = -3 \text{ or } x = 2$$

The solutions are $x = -3$ and $x = 2$.

d) The non-permissible value is $x = 5$.

$$\frac{3}{x-5} = \frac{10}{5x-25},\ x \neq 5$$

$$\frac{3}{x-5} = \frac{5(2)}{5(x-5)}$$

$$\frac{3}{x-5} = \frac{2}{x-5}$$

$$\frac{3}{x-5}(x-5) = \frac{2}{x-5}(x-5)$$

$$3 = 2$$

This equation has no solution.

2. a) Let v represent the original speed, in kilometres per hour. The new time can be represented by $\frac{15}{v-5}$.

b) Let n represent the original number of items. The unit cost can be represented by $\frac{4000}{n+5}$.

c) Let t represent the original time for the journey, in hours. The new speed can be represented by $\frac{50}{2t}$.

d) Let t represent the time for Brad to complete the job. The fraction of the job completed can be represented by $\frac{1}{t+5}$.

3. a) Denominator:

$v - 5 = 0$

$v = 5$

The non-permissible value for this expression is $v = 5$. However, in context, v must be greater than 5. This is because the expression represents time, so it cannot be negative.

b) Denominator:

$n + 5 = 0$

$n = -5$

The non-permissible value for this expression is $n = -5$. However, in context, n must be greater than 0. This is because $n + 5$ represents a number of objects, and n was also a number of objects, and neither of these numbers can be negative.

c) Denominator:

$2t = 0$

$t = 0$

The non-permissible value for this expression is $t = 0$. However, in context, t must be greater than 0, because it represents time.

d) Denominator:

$t + 5 = 0$

$t = -5$

The non-permissible value for this expression is $t = -5$. However, in context, t must be greater than 0, because it represents time.

4. Yes, these are the roots. I know this because these numbers make both sides of the equation equal.

When $x = 3$,

$$\frac{2}{3} = \frac{3-1}{3}$$

$$\frac{2}{3} = \frac{2}{3}$$

LS = RS

When $x = -2$,

$$\frac{2}{-2} = \frac{-2-1}{3}$$

$$\frac{-2}{2} = \frac{-3}{3}$$

$$-1 = -1$$

LS = RS

Also, we can isolate x to determine the roots.

$$\frac{2}{x} = \frac{x-1}{3}$$

$$\frac{2x}{x} = \frac{(x-1)x}{3}$$

$$2 = \frac{x(x-1)}{3}$$

$$2(3) = \frac{3(x^2 - x)}{3}$$

$$6 = x^2 - x$$

$$0 = x^2 - x - 6$$

$$0 = (x-3)(x+2)$$

$x = 3$ or $x = -2$

Therefore, $x = 3$ and $x = -2$ are the only roots of this equation.

5. a) Denominator:

$x - 1 = 0$

$x = 1$

The non-permissible value for this equation is $x = 1$.

$$\frac{x+3}{x-1} = 0$$

$$\frac{(x+3)(x-1)}{x-1} = 0(x-1)$$

$$x + 3 = 0$$

$$x = -3$$

Verify: Let $x = -3$

$$\frac{(-3)+3}{(-3)-1} = 0$$

$$\frac{0}{-4} = 0$$

$$0 = 0$$

LS = RS

b) Denominator:

$x - 1 = 0$

$x = 1$

The non-permissible value for this equation is $x = 1$.

$$\frac{x+3}{x-1} = 2$$

$$\frac{(x+3)(x-1)}{x-1} = 2(x-1)$$

$$x + 3 = 2x - 2$$

$$x - 2x = -2 - 3$$

$$-x = -5$$

$$x = 5$$

Verify: Let $x = 5$

$$\frac{5+3}{5-1} = 2$$

$$\frac{8}{4} = 2$$

$$2 = 2$$

LS = RS

c) Denominator:

$x-1=0$

$x=1$

The non-permissible value for this equation is $x = 1$.

$$\frac{x+3}{x-1}=2x+1$$

$$\frac{(x+3)(x-1)}{x-1}=(2x+1)(x-1)$$

$$x+3=2x^2-x-1$$

$$0=2x^2-2x-4$$

$$0=2(x^2-x-2)$$

$$0=2(x-2)(x+1)$$

$x=2$ or $x=-1$

Verify: Let $x = 2$

$$\frac{2+3}{2-1}=2(2)+1$$

$$\frac{5}{1}=5$$

$$5=5$$

LS = RS

Let $x = -1$

$$\frac{-1+3}{-1-1}=2(-1)+1$$

$$\frac{2}{-2}=-2+1$$

$$-1=-1$$

LS = RS

d) Denominator:

$3x+2=0$ or $5x=0$

$3x=-2$ or $5x=0$

$x=\frac{-2}{3}$ or $x=0$

The non-permissible values for this equation are $x=\frac{-2}{3}$ and $x = 0$.

$$\frac{3}{3x+2}=\frac{6}{5x}$$

$$\frac{3(3x+2)(5x)}{3x+2}=\frac{6(3x+2)(5x)}{5x}$$

$$15x=18x+12$$

$$-3x=12$$

$$x=-4$$

Verify: Let $x = -4$

$$\frac{3}{3(-4)+2}=\frac{6}{5(-4)}$$

$$\frac{3}{-12+2}=\frac{6}{-20}$$

$$\frac{3}{-10}=\frac{6}{-20}$$

$$\frac{-3}{10}=\frac{-3}{10}$$

LS = RS

6. a) Denominator:

$x-3=0$ or $x^2-9=0$

$x-3=0$ or $(x+3)(x-3)=0$

$x-3=0$ or $x+3=0$

$x=3$ or $x=-3$

The non-permissible values for this equation are $x = 3$ and $x = -3$.

$$\frac{1}{x-3}-\frac{6}{x^2-9}=\frac{1}{5}$$

$$\frac{1}{x-3}-\frac{6}{(x+3)(x-3)}=\frac{1}{5}$$

$$\frac{5(x+3)(x-3)}{x-3}-\frac{6(5)(x+3)(x-3)}{(x+3)(x-3)}=\frac{5(x+3)(x-3)}{5}$$

$$5(x+3)-6(5)=(x+3)(x-3)$$

$$5x+15-30=x^2-9$$

$$5x-15=x^2-9$$

$$x^2-5x+6=0$$

$$(x-2)(x-3)=0$$

The solutions to this equation are $x = 2$ and $x = -3$. However, $x = -3$ is a non-permissible value. Therefore, $x = -3$ is an extraneous root, and the only solution is $x = 2$.

Verify: Let $x = 2$

$$\frac{1}{2-3}-\frac{6}{2^2-9}=\frac{1}{5}$$

$$\frac{1}{-1}-\frac{6}{-5}=\frac{1}{5}$$

$$-1-\frac{-6}{5}=\frac{1}{5}$$

$$-1+\frac{6}{5}=\frac{1}{5}$$

$$\frac{1}{5}=\frac{1}{5}$$

LS = RS

b) Denominator:

$z-1=0$

$z=1$

The non-permissible value for this equation is $z = 1$.

$$\frac{z}{z-1}-\frac{1}{z-1}=-4$$

$$\frac{z(z-1)}{z-1}-\frac{z-1}{z-1}=-4(z-1)$$

$$z-1=-4z+4$$

$$5z-5=0$$

$$5(z-1)=0$$

$$z=1$$

The solution to this equation is $z = 1$. However, this is a non-permissible value, so $z = 1$ is an extraneous root. Therefore, this equation has no solution.

c) Denominator:

$x+3=0$ or $4x=0$

$x=-3$ or $x=0$

The non-permissible values for this equation are $x = -3$ and $x = 0$.

$$\frac{5}{x+3}=\frac{2-4x}{4x}+2$$

$$\frac{5}{x+3}=\frac{1-2x}{2x}+2$$

$$\frac{5(x+3)(2x)}{x+3}=\frac{(1-2x)(x+3)(2x)}{2x}+2(x+3)(2x)$$

$$5(2x)=(1-2x)(x+3)+2(x+3)(2x)$$

$$10x=-2x^2-5x+3+4x^2+12x$$

$$10x=2x^2+7x+3$$

$$0=2x^2-3x+3$$

This equation is not factorable so we must use the quadratic formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(3)}}{2(2)}$$

$$x=\frac{3\pm\sqrt{9-24}}{4}$$

$$x=\frac{3\pm\sqrt{-15}}{4}$$

Since $\sqrt{-15}$ doesn't exist, there is no solution to this equation.

d) Denominator:

$m-3=0$ or $m=0$ or $m^2-3m=0$

$m-3=0$ or $m=0$ or $m(m-3)=0$

$m-3=0$ or $m=0$

$m=3$ or $m=0$

The non-permissible values for this equation are $m = 3$ and $m = 0$.

$$\frac{2}{m-3}+\frac{1}{m}=\frac{6}{m^2-3m}$$

$$\frac{2}{m-3}+\frac{1}{m}=\frac{6}{m(m-3)}$$

$$\frac{2m(m-3)}{m-3}+\frac{m(m-3)}{m}=\frac{6m(m-3)}{m(m-3)}$$

$$2m+(m-3)=6$$

$$3m-3=6$$

$$3m-9=0$$

$$3(m-3)=0$$

$$m=3$$

The solution is $m = 3$. However, this is a non-permissible value, so it is an extraneous root. Therefore, this equation has no solution.

7. Let $l = 15$

Denominator:

$w=0$ or $15-w=0$

$w=0$ or $w=15$

The non-permissible values for this equation are $w = 0$ and $w = 15$. In context, w must be greater than 0, because it represents a length.

$$\frac{15}{w}=\frac{w}{15-w}$$

$$\frac{15w(15-w)}{w}=\frac{w^2(15-w)}{15-w}$$

$$15(15-w)=w^2$$

$$225-15w=w^2$$

$$0=w^2+15w-225$$

$$w=\frac{-15\pm\sqrt{15^2-4(1)(-225)}}{2}$$

$$w=\frac{-15\pm\sqrt{225+900}}{2}$$

$$w=\frac{-15\pm\sqrt{1125}}{2}$$

$$w=\frac{-15\pm33.541...}{2}$$

$$w=\frac{-15+33.541...}{2} \text{ or } w=\frac{-15-33.541...}{2}$$

$$w=\frac{18.541...}{2} \quad \text{or } w=\frac{-48.541}{2}$$

$w = 9.270...$ or $w = -24.270...$

However, $w = -24.270...$ is an inadmissible value because it is negative. Therefore, the only solution is $w = 9.270....$ The width of the poster will be about 9.27 m.

8. The non-permissible values are $t = 0$ and $t = -10$. In context, t must be greater than 0, because it represents time.

$$\frac{1}{t}+\frac{1}{t+10}=\frac{1}{15}$$
$$\frac{15t(t+10)}{t}+\frac{15t(t+10)}{t+10}=\frac{15t(t+10)}{15}$$
$$15(t+10)+15t=t(t+10)$$
$$15t+150+15t=t^2+10t$$
$$30t+150=t^2+10t$$
$$0=t^2-20t-150$$
$$t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$t=\frac{-(-20)\pm\sqrt{(20)^2-4(1)(-150)}}{2(1)}$$
$$t=\frac{20\pm\sqrt{400+600}}{2}$$
$$t=\frac{20\pm\sqrt{1000}}{2}$$
$$t=\frac{20\pm10\sqrt{10}}{2}$$
$$t=10\pm5\sqrt{10}$$
$$t=10+5\sqrt{10} \text{ or } t=10-5\sqrt{10}$$
$$t=25.811... \text{ or } t=-5.811...$$

However, $t = -5.811...$ is an inadmissible solution, because it is negative. Therefore, Machine A takes about 26 minutes to fill a case of chocolates, while Machine B takes $t + 10$, or about 36 minutes, to fill a case of chocolates.

9. a) Let v represent Michael's original speed, in kilometres per hour. His slower time is equal to $\frac{9}{v}$.

b) His new time will be equal to $\frac{9}{v+3}$.

c) $\frac{9}{v}=\frac{9}{v+3}+0.25$

d) The non-permissible values for this equation are $v = 0$ and $v = -3$. v represents a speed, so $v > 0$.

e)
$$\frac{9}{v}=\frac{9}{v+3}+0.25$$
$$\frac{9v(v+3)}{v}=\frac{9v(v+3)}{v+3}+0.25v(v+3)$$
$$9(v+3)=9v+0.25v(v+3)$$
$$9v+27=9v+0.25v^2+0.75v$$
$$0=9v+0.25v^2+0.75v-9v-27$$
$$0=0.25v^2+0.75v-27$$
$$0=0.25(v^2+3v-108)$$
$$0=0.25(v+12)(v-9)$$
$$v=-12 \text{ or } v=9$$

However, $v = -12$ is an inadmissible solution, because it is negative.
Verify: $v = 9$

$$\frac{9}{(9)}=\frac{9}{(9)+3}+0.25$$
$$\frac{9}{9}=\frac{9}{12}+0.25$$
$$1=0.75+0.25$$
$$1=1$$
$$LS=RS$$

Therefore, the slower speed was 9 km/h, and the faster speed is 12 km/h.

10. Let t represent the time it would take for both Susan and Jacqueline to paint a room if they worked together, in hours.
The non-permissible value for this equation is $t = 0$. In context, t must be greater than 0, because it represents time.

$$\frac{1}{3}+\frac{1}{2}=\frac{1}{t}$$
$$\frac{2}{6}+\frac{3}{6}=\frac{1}{t}$$
$$\frac{5}{6}=\frac{1}{t}$$
$$\frac{5t}{6}=\frac{t}{t}$$
$$\frac{5t}{6}=1$$
$$5t=6$$
$$t=\frac{6}{5}$$

$\frac{6}{5}$ hours = $\frac{6}{5}\cdot 60$ minutes
= 72 minutes

It would take Susan and Jacqueline 72 min or 1 h 12 min to paint a room if they worked together.

11. Let t represent the time it takes for Caleb to complete his chores by himself, in minutes. Let $t + 36$ represent the time it takes for Jonah to complete his chores by himself, in minutes.
The non-permissible values for this equation are $t = 0$ and $t = -36$. However, in context, t must be greater than 0, because it represents time.

$$\frac{1}{t}+\frac{1}{t+36}=\frac{1}{80}$$
$$\frac{80t(t+36)}{t}+\frac{80t(t+36)}{t+36}=\frac{80t(t+36)}{80}$$
$$80(t+36)+80t=t(t+36)$$
$$80t+2880+80t=t^2+36t$$
$$160t+2880=t^2+36t$$
$$0=t^2+36t-160t-2880$$
$$0=t^2-124t-2880$$
$$0=(t-144)(t+20)$$
$$t=144 \text{ or } t=-20$$

However, $t = -20$ is an inadmissible solution, because it is negative.
Verify: $t = 144$

$$\frac{1}{(144)}+\frac{1}{(144)+36}=\frac{1}{80}$$
$$\frac{1}{144}+\frac{1}{180}=\frac{1}{80}$$
$$\frac{5}{144(5)}+\frac{4}{180(4)}=\frac{1}{80}$$
$$\frac{9}{720}=\frac{1}{80}$$
$$\frac{1}{80}=\frac{1}{80}$$
$$LS=RS$$

Caleb takes 144 min to complete his chores alone.

12. Let t represent the time it would take for Enzo to build an outdoor patio by himself, in hours. Let $t + 9$ represent the time it would tale for Rahj to build an outdoor patio by himself, in hours.
The non-permissible values for this equation are $t = 0$, and $t = -9$. However, in context, t must be greater than 0, because it represents a time.

$$\frac{1}{t}+\frac{1}{t+9}=\frac{1}{20}$$
$$\frac{20t(t+9)}{t}+\frac{20t(t+9)}{t+9}=\frac{20t(t+9)}{20}$$
$$20(t+9)+20t=t(t+9)$$
$$20t+180+20t=t^2+9t$$
$$40t+180=t^2+9t$$
$$0=t^2+9t-40t-180$$
$$0=t^2-31t-180$$
$$0=(t+5)(t-36)$$
$$t=-5 \text{ or } t=36$$

However. $t = -5$ is an inadmissible solution, because it is negative.

Verify: Let $t = 36$

$$\frac{1}{(36)}+\frac{1}{(36)+9}=\frac{1}{20}$$
$$\frac{1}{36}+\frac{1}{45}=\frac{1}{20}$$
$$\frac{5}{36(5)}+\frac{4}{45(4)}=\frac{1}{20}$$
$$\frac{9}{180}=\frac{1}{20}$$
$$\frac{1}{20}=\frac{1}{20}$$
$$LS=RS$$

It would take Enzo 36 h and Rahj 45 h to build outdoor patios by themselves.

13. Let v represent her original speed in kilometres per hour, and let $v + 3$ represent her new speed in kilometres per hour.
The non-permissible values for this equation are $v = 0$ and $v = -3$. In context, v must be greater than 0, because v represents speed.

$$\frac{4}{v}+\frac{6}{v+3}=\frac{10}{\frac{10}{0.6}}$$
$$\frac{4}{v}+\frac{6}{v+3}=10\cdot\frac{0.6}{10}$$
$$\frac{4}{v}+\frac{6}{v+3}=0.6$$
$$\frac{4v(v+3)}{v}+\frac{6v(v+3)}{v+3}=0.6v(v+3)$$
$$4(v+3)+6v=0.6v(v+3)$$
$$4v+12+6v=0.6v^2+1.8v$$
$$10v+12=0.6v^2+1.8v$$
$$0=0.6v^2-8.2v-12$$
$$0=0.2(3v^2-41v-60)$$
$$0=0.2(3v+4)(v-15)$$
$$v=-\frac{4}{3} \text{ or } v=15$$

However, $v = -\frac{4}{3}$ is an inadmissible solution, because it is negative.

Verify: Let $v = 15$

$$\frac{4}{(15)}+\frac{6}{(15)+3}=0.6$$
$$\frac{4}{15}+\frac{6}{18}=0.6$$
$$\frac{4(6)}{15(6)}+\frac{6(5)}{18(5)}=0.6$$
$$\frac{24+30}{90}=0.6$$
$$\frac{54}{90}=0.6$$
$$0.6=0.6$$
$$LS=RS$$

Tara will need to run $v + 3$, or 18 km/h for the remaining 6 km to finish in 36 minutes.

14. Let v represent Lydia's average speed, in kilometres per hour.
The non-permissible values for this equation are $v = 0$ and $v = -24$. In context, v must be greater than 0, because it represents a speed.

$$\frac{189}{v}=\frac{189}{v+24}+\frac{1}{3}$$
$$\frac{189v(v+24)}{v}=\frac{189v(v+24)}{v+24}+\frac{v(v+24)}{3}$$
$$189(v+24)=189v+\frac{v^2+24v}{3}$$
$$189v+4536=189v+\frac{1}{3}v^2+8v$$
$$0=189v+\frac{1}{3}v^2+8v-189v-4536$$
$$0=\frac{1}{3}v^2+8v-4536$$
$$v=\frac{-8\pm\sqrt{(8)^2-\left(4\cdot\frac{1}{3}\cdot-4536\right)}}{2\cdot\frac{1}{3}}$$
$$v=\frac{-8\pm\sqrt{64+6048}}{\frac{2}{3}}$$
$$v=\left(-8\pm\sqrt{6112}\right)\cdot\frac{3}{2}$$
$$v=\left(-8\pm4\sqrt{382}\right)\cdot\frac{3}{2}$$
$$v=-12\pm6\sqrt{382}$$
$$v=-12+6\sqrt{382} \text{ or } v=-12-6\sqrt{382}$$
$$v=105.268... \quad \text{or } v=-129.268...$$

However, $v = -129.268$ is an inadmissible solution, because it is negative. Therefore, Lydia's average speed is about 105 km/h.

15. Let x represent the number of bicycles ordered, and let $x - 3$ represent the number of bicycles sold.
The non-permissible values for this equation are $x = 0$ and $x = 3$. In context, x must be greater than 3, because at least 3 bicycles were ordered.

$$300=\frac{70500}{x-3}-\frac{60000}{x}$$
$$300x(x-3)=\frac{70500x(x-3)}{x-3}-\frac{60000x(x-3)}{x}$$
$$300x(x-3)=70500x-60000(x-3)$$
$$300x^2-900x=70500x-60000x+180000$$
$$300x^2-900x=10500x+180000$$
$$300x^2-900x-10500x-180000=0$$
$$300x^2-11400x-180000=0$$
$$300(x^2-38x-600)=0$$
$$300(x-50)(x+12)=0$$
$$x=50 \text{ or } x=-12$$

However, $-12 < 3$, or $x = -12$ is an inadmissible solution.

Verify: Let $x = 50$

$$300=\frac{70500}{(50)-3}-\frac{60000}{50}$$
$$300=\frac{70500}{47}-\frac{60000}{50}$$
$$300=1500-1200$$
$$300=300$$
$$LS=RS$$

Sarah ordered 50 bicycles.

16. Let x represent the cost of each high performance tire, and let $x - 3$ represent the cost of each regular tire.
The non-permissible values for the equation are $x = 0$ and $x = 3$. In context, x must be greater than 3, because the cost of each high-performance tire is \$3 more than the cost of each regular tire.

$$330=\frac{1200}{x}+\frac{3000}{x-3}$$
$$330x(x-3)=\frac{1200x(x-3)}{x}+\frac{3000x(x-3)}{x-3}$$
$$330x(x-3)=1200(x-3)+3000x$$
$$330x^2-990x=1200x-3600+3000x$$
$$330x^2-990x=4200x-3600$$
$$330x^2-990x-4200x+3600=0$$
$$330x^2-5190x+3600=0$$
$$30(11x^2-173x+120)=0$$
$$30(11x-8)(x-15)=0$$
$$x=\frac{8}{11} \text{ or } x=15$$

However, $\frac{8}{11} < 3$, so $x = \frac{8}{11}$ is an inadmissible solution.

Verify: Let $x = 15$

$$330 = \frac{1200}{(15)} + \frac{3000}{(15)-3}$$

$$330 = \frac{1200}{15} + \frac{3000}{12}$$

$$330 = 80 + 250$$

$$330 = 330$$

$$LS = RS$$

The cost of each high performance tire is \$15.

17. Let x represent the number of children who went on the field trip, and let $x + 5$ represent the number of children who were supposed to go on the field trip.
The non-permissible values for this equation are $x = 0$ and $x = -5$. In context, x must be greater than 0, because x represents a number of objects.

$$\frac{5400}{x+5} + 15 = \frac{5400}{x}$$

$$\frac{5400x(x+5)}{x+5} + 15x(x+5) = \frac{5400x(x+5)}{x}$$

$$5400x + 15x(x+5) = 5400(x+5)$$

$$5400x + 15x^2 + 75x = 5400x + 27000$$

$$5400x + 15x^2 + 75x - 5400 - 27000 = 0$$

$$15x^2 + 75x - 27000 = 0$$

$$15(x^2 + 5x - 1800) = 0$$

$$15(x+45)(x-40) = 0$$

$$x = -45 \text{ or } x = 40$$

However, -45 is less than 0, so $x = -45$ is an inadmissible solution.
Verify: Let $x = 40$

$$\frac{5400}{(40)+5} + 15 = \frac{5400}{(40)}$$

$$\frac{5400}{45} + 15 = \frac{5400}{40}$$

$$\frac{5400}{45} + 15 = \frac{5400}{40}$$

$$120 + 15 = 135$$

$$135 = 135$$

$$LS = RS$$

40 students went on the field trip.

18. Let x represent the number of necklaces that Cheyenne will make, and let $x - 3$ represent the number of necklaces that Cheyenne will sell.
The non-permissible values for this equation are $x = 0$ and $x = 3$. In context, x must be greater than 3, because Cheyenne keeps 3 necklaces for her family.

$$12.5 = \frac{250}{x-3} - \frac{75}{x}$$

$$12.5x(x-3) = \frac{250x(x-3)}{x-3} - \frac{75x(x-3)}{x}$$

$$12.5x(x-3) = 250x - 75(x-3)$$

$$12.5x^2 - 37.5x = 250x - 75x + 225$$

$$12.5x^2 - 37.5x = 175x + 225$$

$$12.5x^2 - 37.5x - 175x - 225 = 0$$

$$12.5x^2 - 212.5x - 225 = 0$$

$$12.5(x^2 - 17x - 18) = 0$$

$$12.5(x-18)(x+1) = 0$$

$$x = 18 \text{ or } x = -1$$

However, $-1 < 3$, so $x = -1$ is an inadmissible solution.

Verify: Let $x = 18$

$$12.5 = \frac{250}{(18)-3} - \frac{75}{18}$$

$$12.5 = \frac{250}{15} - \frac{75}{18}$$

$$12.5 = \frac{250(6)}{15(6)} - \frac{75(5)}{18(5)}$$

$$12.5 = \frac{1500-375}{90}$$

$$12.5 = \frac{1125}{90}$$

$$12.5 = 12.5$$

$$LS = RS$$

Cheyenne needs to make 18 necklaces in order to make a profit of \$12.50 per necklace.

19. Erin and Dawn are sisters. It takes Erin 20 min longer to wash down their backyard fence than it takes Dawn. Together, it takes the sisters 24 min to finish the fence. How long would it take each sister to complete the task on her own?
Solution: Let t represent the amount of time it would take for Dawn to wash down the backyard fence, in minutes, and let $t + 20$ represent the amount of time it would take for Erin to wash down the backyard fence, in minutes.
The non-permissible values for this equation are $t = 0$ and $t = -20$. In context, t must be greater than 0, because it represents a time.

$$\frac{1}{t}+\frac{1}{t+20}=\frac{1}{24}$$

$$\frac{24t(t+20)}{t}+\frac{24t(t+20)}{t+20}=\frac{24t(t+20)}{24}$$

$$24(t+20)+24t=t(t+20)$$

$$24t+480+24t=t^2+20t$$

$$48t+480=t^2+20t$$

$$0=t^2+20t-48t-480$$

$$0=t^2-28t-480$$

$$0=(t-40)(t+12)$$

$$t=40 \text{ or } t=-12$$

However, –12 is less than 0, so $t = -12$ is an inadmissible solution.

Verify: Let $t = 40$

$$\frac{1}{(40)}+\frac{1}{(40)+20}=\frac{1}{24}$$

$$\frac{1}{40}+\frac{1}{60}=\frac{1}{24}$$

$$\frac{3}{40(3)}+\frac{2}{60(2)}=\frac{1}{24}$$

$$\frac{5}{120}=\frac{1}{24}$$

$$\frac{1}{24}=\frac{1}{24}$$

LS = RS

It would take Dawn 40 min, and Erin 60 min to wash down the fence individually.

20. Let x represent the amount of time it would take for Pietr to load a truck by himself, and let $x - 3$ represent the amount of time it would take for Theresa to load a truck by herself.

The non-permissible values for this equation are $x = 0$ and $x = 3$. In context, x must be greater than 3, because Pietr takes 3 min more than Theresa to load a truck.

$$\frac{6}{x-3}+\frac{8}{x}=1$$

$$\frac{6x(x-3)}{x-3}+\frac{8x(x-3)}{x}=1x(x-3)$$

$$6x+8(x-3)=x(x-3)$$

$$6x+8x-24=x^2-3x$$

$$14x-24=x^2-3x$$

$$0=x^2-3x-14x+24$$

$$0=x^2-17x+24$$

$$x=\frac{-(-17)\pm\sqrt{(-17)^2-4(1)(24)}}{2(1)}$$

$$x=\frac{17\pm\sqrt{289-96}}{2}$$

$$x=\frac{17\pm\sqrt{193}}{2}$$

$$x=\frac{17+\sqrt{193}}{2} \text{ or } x=\frac{17-\sqrt{193}}{2}$$

$x = 15.446...$ or $x = 1.553...$

However, 1.553 is less than 3, so $x = 1553...$ is an inadmissible solution.

It would take Pietr 15.446...min, or about 15 min 27 s to load the truck on his own.

Math in Action, page 261

• I estimate that Niobe must ride at 30 km/h to finish the trip at a speed of 25 km/h.

$$\text{Total time}=\frac{50 \text{ km}}{20 \text{ km/h}}+\frac{50 \text{ km}}{20 \text{ km/h}}$$

Total time = 4.5 h

$$\text{Average speed}=\frac{\text{Total distance}}{\text{Total time}}$$

$$\text{Average speed}=\frac{100 \text{ km}}{4.5 \text{ h}}$$

Average speed = 22.222... km/h

- New estimate: 33 km/h

$$\text{Total time}=\frac{50 \text{ km}}{20 \text{ km/h}}+\frac{50 \text{ km}}{33 \text{ km/h}}$$

Total time = 4.015... h

$$\text{Average speed}=\frac{100 \text{ km}}{4.015... \text{ h}}$$

Average speed = 24.905... km/h

- New estimate: 34 km/h

$$\text{Total time}=\frac{50 \text{ km}}{20 \text{ km/h}}+\frac{50 \text{ km}}{33.5 \text{ km/h}}$$

Total time = 3.992... h

$$\text{Average speed}=\frac{100 \text{ km}}{3.992... \text{ h}}$$

Average speed = 25.046... km/h

Niobe needs to finish her trip at a speed of about 33.5 km/h.

$$H = \frac{2ab}{a+b}$$
$$25 = \frac{2b(20)}{20+b},\ b \neq -20$$
$$25(20+b) = 40b$$
$$500 + 25b = 40b$$
$$500 = 15b$$
$$\frac{100}{3} = b$$
$$33.333... = b$$

Niobe needs to finish her trip at a speed of 33.3 km/h.

Chapter Self-Test, page 262

1. a) $\frac{-38a}{16a^2} = \frac{2a(-19)}{2a(8a)},\ a \neq 0$

$\frac{-38a}{16a^2} = \frac{-19}{8a},\ a \neq 0$

b) $\frac{15x(2-x)}{x-2} = \frac{-15x(x-2)}{x-2},\ x \neq 2$

$\frac{15x(2-x)}{x-2} = -15x,\ x \neq 2$

c) $\frac{12x^2+24x}{6x^2} = \frac{12x(x+2)}{6x^2},\ x \neq 0$

$\frac{12x^2+24x}{6x^2} = \frac{6x(2)(x+2)}{6x(x)}$

$\frac{12x^2+24x}{6x^2} = \frac{2(x+2)}{x},\ x \neq 0$

d) $\frac{b-1}{15-15b} = \frac{b-1}{15(1-b)},\ b \neq 1$

$\frac{b-1}{15-15b} = \frac{-(1-b)}{15(1-b)}$

$\frac{b-1}{15-15b} = \frac{-1}{15},\ b \neq 1$

2. a) $\frac{12(2b-1)}{2b} \cdot \frac{1}{6b(2b-1)}$

$= \frac{6(2b-1)}{b} \cdot \frac{1}{6b(2b-1)},\ b \neq 0,\ \frac{1}{2}$

$= \frac{6(2b-1)}{6b^2(2b-1)}$

$= \frac{1}{b^2},\ b \neq 0,\ \frac{1}{2}$

b) $\frac{3x^2-3x}{10} \div \frac{x-1}{5x} = \frac{3x(x-1)}{10} \div \frac{x-1}{5x},\ x \neq 0, 1$

$\frac{3x^2-3x}{10} \div \frac{x-1}{5x} = \frac{3x(x-1)}{10} \cdot \frac{5x}{x-1}$

$\frac{3x^2-3x}{10} \div \frac{x-1}{5x} = \frac{15x^2(x-1)}{10(x-1)}$

$\frac{3x^2-3x}{10} \div \frac{x-1}{5x} = \frac{5(3x^2)(x-1)}{5(2)(x-1)}$

$\frac{3x^2-3x}{10} \div \frac{x-1}{5x} = \frac{3x^2}{2},\ x \neq 0, 1$

c) $\frac{3-x}{x} \div \frac{9-x^2}{x^3} = \frac{3-x}{x} \div \frac{(3-x)(3+x)}{x^3},\ x \neq -3, 0, 3$

$\frac{3-x}{x} \div \frac{9-x^2}{x^3} = \frac{3-x}{x} \cdot \frac{x^3}{(3-x)(3+x)}$

$\frac{3-x}{x} \div \frac{9-x^2}{x^3} = \frac{x^3(3-x)}{x(3-x)(3+x)}$

$\frac{3-x}{x} \div \frac{9-x^2}{x^3} = \frac{x^2}{3+x},\ x \neq 0, 3, -3$

d) $\frac{12a}{a-3} \cdot \frac{2a-6}{-6a} = \frac{12a}{a-3} \cdot \frac{2(a-3)}{-6a},\ a \neq 0, 3$

$\frac{12a}{a-3} \cdot \frac{2a-6}{-6a} = \frac{12a}{a-3} \cdot \frac{-(a-3)}{3a}$

$\frac{12a}{a-3} \cdot \frac{2a-6}{-6a} = \frac{-12a(a-3)}{3a(a-3)}$

$\frac{12a}{a-3} \cdot \frac{2a-6}{-6a} = -4,\ a \neq 0, 3$

3. a) $\frac{4}{b} - \frac{b+6}{2b} = \left(\frac{4}{b} \cdot \frac{2}{2}\right) - \frac{b+6}{2b},\ b \neq 0$

$\frac{4}{b} - \frac{b+6}{2b} = \frac{8}{2b} - \frac{b+6}{2b}$

$\frac{4}{b} - \frac{b+6}{2b} = \frac{8-b-6}{2b}$

$\frac{4}{b} - \frac{b+6}{2b} = \frac{2-b}{2b},\ b \neq 0$

b) $\frac{-6x^3}{2x(x+1)} + 3x = \frac{-3x^2}{(x+1)} + 3x,\ x \neq -1, 0$

$\frac{-6x^3}{2x(x+1)} + 3x = \frac{-3x^2}{(x+1)} + \left(3x \cdot \frac{x+1}{x+1}\right)$

$\frac{-6x^3}{2x(x+1)} + 3x = \frac{-3x^2}{(x+1)} + \frac{3x(x+1)}{x+1}$

$\frac{-6x^3}{2x(x+1)} + 3x = \frac{3x(x+1)-3x^2}{x+1}$

$\frac{-6x^3}{2x(x+1)} + 3x = \frac{3x^2+3x-3x^2}{x+1}$

$\frac{-6x^3}{2x(x+1)} + 3x = \frac{3x}{x+1},\ x \neq -1, 0$

c) $\dfrac{6x+5}{1-4x^2}-\dfrac{15}{2+4x}$

$=\dfrac{6x+5}{(1+2x)(1-2x)}-\dfrac{15}{2(1+2x)}, x\neq\dfrac{-1}{2},\dfrac{1}{2}$

$=\left(\dfrac{6x+5}{(1+2x)(1-2x)}\cdot\dfrac{2}{2}\right)-\left(\dfrac{15}{2(1+2x)}\cdot\dfrac{1-2x}{1-2x}\right)$

$=\dfrac{2(6x+5)}{2(1+2x)(1-2x)}-\dfrac{15(1-2x)}{2(1+2x)(1-2x)}$

$=\dfrac{12x+10}{2(1+2x)(1-2x)}-\dfrac{15-30x}{2(1+2x)(1-2x)}$

$=\dfrac{12x+10-15+30x}{2(1+2x)(1-2x)}$

$=\dfrac{42x-5}{2(1+2x)(1-2x)}, x\neq\dfrac{-1}{2},\dfrac{1}{2}$

d) $\dfrac{3}{x-6}-\dfrac{1}{2x-12}=\dfrac{3}{x-6}-\dfrac{1}{2(x-6)}, x\neq 6$

$\dfrac{3}{x-6}-\dfrac{1}{2x-12}=\left(\dfrac{3}{x-6}\cdot\dfrac{2}{2}\right)-\dfrac{1}{2(x-6)}$

$\dfrac{3}{x-6}-\dfrac{1}{2x-12}=\dfrac{6}{2(x-6)}-\dfrac{1}{2(x-6)}$

$\dfrac{3}{x-6}-\dfrac{1}{2x-12}=\dfrac{5}{2(x-6)}, x\neq 6$

4. a) The non-permissible value of this equation is $x = 2$.

$$\frac{3x+2}{x-2}+1=-5$$
$$\frac{3x+2(x-2)}{x-2}+1(x-2)=-5(x-2)$$
$$3x+2+x-2=-5x+10$$
$$3x+x+5x+2-2-10=0$$
$$9x-10=0$$
$$9x=10$$
$$x=\frac{10}{9}$$

Verify: Let $x=\dfrac{10}{9}$

$$\frac{3\left(\frac{10}{9}\right)+2}{\left(\frac{10}{9}\right)-2}+1=-5$$
$$\frac{\frac{16}{3}}{\frac{-8}{9}}+1=-5$$
$$\left(\frac{16}{3}\cdot\frac{-9}{8}\right)+1=-5$$
$$-6+1=-5$$
$$-5=-5$$
$$LS=RS$$

The solution to this equation is $x=\dfrac{10}{9}$.

b) The non-permissible value of this equation is $a = 0$.

$$\frac{3}{a}-6=\frac{5a}{2a^2}$$
$$\frac{3}{a}-6=\frac{5}{2a}$$
$$\frac{3(2a)}{a}-6(2a)=\frac{5(2a)}{2a}$$
$$6-12a=5$$
$$6-5-12a=0$$
$$1-12a=0$$
$$-12a=-1$$
$$a=\frac{1}{12}$$

Verify: Let $a=\dfrac{1}{12}$

$$\frac{3}{\left(\frac{1}{12}\right)}-6=\frac{5\left(\frac{1}{12}\right)}{2\left(\frac{1}{12}\right)^2}$$
$$\frac{3}{\frac{1}{12}}-6=\frac{\frac{5}{12}}{2\left(\frac{1}{144}\right)}$$
$$\frac{3}{\frac{1}{12}}-6=\frac{\frac{5}{12}}{\frac{1}{72}}$$
$$(3\cdot 12)-6=\left(\frac{5}{12}\cdot 72\right)$$
$$36-6=30$$
$$30=30$$
$$LS=RS$$

The solution to this equation is $a=\dfrac{1}{12}$.

c) The non-permissible value of this equation is $a = 1$.

$$\frac{a}{a-1}-\frac{2}{1-a}=7$$
$$\frac{a}{a-1}-\frac{2}{-(a-1)}=7$$
$$\frac{a}{a-1}+\frac{2}{a-1}=7$$
$$\frac{a+2}{a-1}=7$$
$$\frac{(a+2)(a-1)}{a-1}=7(a-1)$$
$$a+2=7a-7$$
$$-6a=-9$$
$$a=\frac{3}{2}$$

Verify: Let $a = \frac{3}{2}$

$$\frac{\left(\frac{3}{2}\right)}{\left(\frac{3}{2}\right)-1} - \frac{2}{1-\left(\frac{3}{2}\right)} = 7$$

$$\frac{\frac{3}{2}}{\frac{1}{2}} - \frac{2}{\frac{-1}{2}} = 7$$

$$\left(\frac{3}{2}\cdot 2\right) - (2\cdot -2) = 7$$

$$3-(-4) = 7$$

$$3+4 = 7$$

$$7 = 7$$

$$LS = RS$$

The solution to this equation is $a = \frac{3}{2}$.

d) The non-permissible value of this equation is $x = -1$.

$$5 - \frac{2x-1}{x+1} = -\frac{2x-1}{x+1}$$

$$5(x+1) - \frac{(2x-1)(x+1)}{x+1} = -\frac{(2x-1)(x+1)}{x+1}$$

$$5x+5-(2x-1) = -(2x-1)$$

$$5x+5-2x+1 = -2x+1$$

$$3x+6 = -2x+1$$

$$5x = -5$$

$$x = -1$$

The root of this equation is $x = -1$. However, this is a non-permissible value, making this root extraneous. Therefore, this equation has no solution.

5. Let v represent the speed of the tailwind. The non-permissible values for this equation are $v = 220$ and $v = -220$. In context, $-220 < v < 220$, because both rational expressions need to be positive.

$$\frac{275}{220-v} + \frac{275}{220+v} = 5.05$$

$$\frac{275(220-v)(220+v)}{220-v} + \frac{275(220-v)(220+v)}{220+v} = 5.05(220-v)(220+v)$$

$$275(220+v) + 275(220-v) = 5.05(48\,400-v^2)$$

$$60\,500 + 275v + 60\,500 - 275v = 244\,420 - 5.05v^2$$

$$121\,000 = 244\,420 - 5.05v^2$$

$$5.05v^2 = 123\,420$$

$$v^2 = 24439.603...$$

$$v = \pm 156.331...$$

However, $-156.331... < 0$, so $v = -156.331...$ is an inadmissible solution.
The speed of the wind was about 156.33 km/h.

6. Let x represent the number of sunglasses sold, and let $x + 10$ represent the number of glasses imported. The non-permissible values for this equation are $x = 0$ and $x = -10$. In context, x must be greater than 0, because it represents a number of objects.

$$5 = \frac{8900}{x} - \frac{4500}{x+10}$$

$$5x(x+10) = \frac{8900x(x+10)}{x} - \frac{4500x(x+10)}{x+10}$$

$$5x(x+10) = 8900(x+10) - 4500x$$

$$5x^2 + 50x = 8900x + 89\,000 - 4500x$$

$$5x^2 + 50x = 4400x + 89\,000$$

$$5x^2 + 50x - 4400x - 89\,000 = 0$$

$$5x^2 - 4350x - 89\,000 = 0$$

$$5(x^2 - 870x - 17800) = 0$$

$$5(x+20)(x-890) = 0$$

$$x = -20 \text{ or } x = 890$$

However $-20 < 0$, or $x = -20$ is an inadmissible solution.

Verify: Let $x = 890$

$$5 = \frac{8900}{(890)} - \frac{4500}{(890)+10}$$

$$5 = \frac{8900}{890} - \frac{4500}{900}$$

$$5 = 10 - 5$$

$$5 = 5$$

$$LS = RS$$

The retailer sold 890 pairs of sunglasses.

7. Let x represent the rate of calories burned per hours while cycling, and let $x + 125$ represent the rate of calories burned per hour while running. The non-permissible values for this equation are $x = 0$ and $x = -125$. In context, x must be greater than 0, because Kathryn is burning calories, not gaining them.

$$3 = \frac{690}{x} + \frac{1260}{x+125}$$

$$3x(x+125) = \frac{690x(x+125)}{x} + \frac{1260x(x+125)}{x+125}$$

$$3x(x+125) = 690(x+125) + 1260x$$

$$3x^2 + 375x = 690x + 86\,250 + 1260x$$

$$3x^2 + 375x = 1950x + 86\,250$$

$$3x^2 + 375x - 1950x - 86\,250 = 0$$

$$3x^2 - 1575x - 86\,250 = 0$$

$$3(x^2 - 525x - 28750) = 0$$

$$3(x-575)(x+20) = 0$$

$$x = 575 \text{ or } x = -20$$

However, $-20 < 0$, so $x = -20$ is an inadmissible solution.

Verify: Let $x = 575$

$$3 = \frac{690}{(575)} + \frac{1260}{(575)+125}$$
$$3 = \frac{690}{575} + \frac{1260}{700}$$
$$3 = \frac{690(28)}{575(28)} + \frac{1260(23)}{700(23)}$$
$$3 = \frac{690(28)+1260(23)}{16100}$$
$$3 = \frac{19320+28980}{16100}$$
$$3 = \frac{48300}{16100}$$
$$3 = 3$$

LS = RS

Kathryn burns 575 Cal per hour while cycling.

8. Let x represent the amount of time it would take for John to shovel the driveway by himself, in minutes, and let $x + 25$ represent the amount of time it would take for Colin to shovel the driveway by himself, in minutes. The non-permissible values for this equation are $x = 0$ and $x = -25$. In context, x must be greater than 0 because x represents a time.

$$\frac{1}{x} + \frac{1}{x+25} = \frac{1}{30}$$
$$\frac{30x(x+25)}{x} + \frac{30x(x+25)}{x+25} = \frac{30x(x+25)}{30}$$
$$30(x+25)+30x = x(x+25)$$
$$30x+750+30x = x^2+25x$$
$$60x+750 = x^2+25x$$
$$0 = x^2+25x-60x-750$$
$$0 = x^2-35x-750$$
$$0 = (x-50)(x+15)$$
$$x = 50 \text{ or } x = -15$$

However, $-15 < 0$, or $x = -15$ is an inadmissible solution.

Verify: Let $x = 50$

$$\frac{1}{(50)} + \frac{1}{(50)+25} = \frac{1}{30}$$
$$\frac{1}{50} + \frac{1}{75} = \frac{1}{30}$$
$$\frac{3}{50(3)} + \frac{2}{75(2)} = \frac{1}{30}$$
$$\frac{5}{150} = \frac{1}{30}$$
$$\frac{1}{30} = \frac{1}{30}$$

LS = RS

It would take 50 min for John to shovel the driveway by himself, and it would take 75 min for Colin to shovel the driveway by himself.

Chapter Review, page 265

1. a) e.g., $\frac{12}{8x+4} = \frac{12}{8x+4} \cdot \frac{-1}{-1}$

$$\frac{12}{8x+4} = \frac{-12}{-8x-4}, x \neq \frac{-1}{2}$$
$$\frac{12}{8x+4} = \frac{4(3)}{4(2x+1)}$$
$$\frac{12}{8x+4} = \frac{3}{2x+1}, x \neq \frac{-1}{2}$$

b) e.g., $\frac{x-3}{x+3} = \frac{x-3}{x+3} \cdot \frac{2}{2}$

$$\frac{x-3}{x+3} = \frac{2x-6}{2x+6}, x \neq -3$$

$$\frac{x-3}{x+3} = \frac{x-3}{x+3} \cdot \frac{x+3}{x+3}$$
$$\frac{x-3}{x+3} = \frac{x^2-9}{(x+3)^2}, x \neq -3$$

c) e.g., $\frac{8a^2}{2a-6} = \frac{2(4a^2)}{2(a-3)}$

$$\frac{8a^2}{2a-6} = \frac{4a^2}{a-3}, a \neq 3$$
$$\frac{8a^2}{2a-6} = \frac{8a^2}{2a-6} \cdot \frac{4}{4}$$
$$\frac{8a^2}{2a-6} = \frac{32a^2}{8a-24}, a \neq 3$$

d) e.g., $\frac{16m^3-12m^2}{6m^4} = \frac{2(8m^3-6m^2)}{2(3m^4)}$

$$\frac{16m^3-12m^2}{6m^4} = \frac{8m^3-6m^2}{3m^4}, m \neq 0$$

$$\frac{16m^3-12m^2}{6m^4} = \frac{m^2(16m-12)}{m^2(6m^2)}$$
$$\frac{16m^3-12m^2}{6m^4} = \frac{16m-12}{6m^2}, m \neq 0$$

2. The rational expression $\frac{320}{w}$ represents the length of the property. e.g., Any non-positive value of w is non-permissible because width cannot be negative.

3. a) $\frac{6n}{14n^3} = \frac{2n(3)}{2n(7n^2)}, n \neq 0$

$$\frac{6n}{14n^3} = \frac{3}{7n^2}, n \neq 0$$

b) $\frac{-15y^4}{20y(y-2)} = \frac{5y(-3y^3)}{5y(4)(y-2)}, y \neq 0, 2$

$$\frac{-15y^4}{20y(y-2)} = \frac{-3y^3}{4(y-2)}, y \neq 0, 2$$

c) $\dfrac{36x^3}{20x^3(2x-1)} = \dfrac{4x^3(9)}{4x^3(5)(2x-1)}, x \neq 0, \dfrac{1}{2}$

$\dfrac{36x^3}{20x^3(2x-1)} = \dfrac{9}{5(2x-1)}, x \neq 0, \dfrac{1}{2}$

d) $\dfrac{4(a-2)(a+1)}{12a(a+1)} = \dfrac{4(a+1)(a-2)}{4(a+1)(3a)}, a \neq -1, 0$

$\dfrac{4(a-2)(a+1)}{12a(a+1)} = \dfrac{a-2}{3a}, a \neq -1, 0$

4. a) $\dfrac{x-6}{30-5x} = \dfrac{x-6}{5(6-x)}, x \neq 6$

$\dfrac{x-6}{30-5x} = \dfrac{x-6}{5(-1)(x-6)}$

$\dfrac{x-6}{30-5x} = \dfrac{-1}{5}, x \neq 6$

b) $\dfrac{18a^2}{12a-24a^2} = \dfrac{18a^2}{12a(1-2a)}, a \neq 0, \dfrac{1}{2}$

$\dfrac{18a^2}{12a-24a^2} = \dfrac{6a(3a)}{6a(2)(1-2a)}$

$\dfrac{18a^2}{12a-24a^2} = \dfrac{3a}{2(1-2a)}, a \neq 0, \dfrac{1}{2}$

c) $\dfrac{36y^2-1}{12y^2-2y} = \dfrac{(6y+1)(6y-1)}{2y(6y-1)}, y \neq 0, \dfrac{1}{6}$

$\dfrac{36y^2-1}{12y^2-2y} = \dfrac{6y+1}{2y}, y \neq 0, \dfrac{1}{6}$

d) $\dfrac{7a^3-28a}{15a+30} = \dfrac{7a(a^2-4)}{15(a+2)}, a \neq -2$

$\dfrac{7a^3-28a}{15a+30} = \dfrac{7a(a+2)(a-2)}{15(a+2)}$

$\dfrac{7a^3-28a}{15a+30} = \dfrac{7a(a-2)}{15}, a \neq -2$

5. a) $\dfrac{1}{4x^2} \cdot \dfrac{22x}{5} = \dfrac{22x}{20x^2}, x \neq 0$

$\dfrac{1}{4x^2} \cdot \dfrac{22x}{5} = \dfrac{2x(11)}{2x(10x)}$

$\dfrac{1}{4x^2} \cdot \dfrac{22x}{5} = \dfrac{11}{10x}, x \neq 0$

b) $\left(\dfrac{15a}{3a-6}\right)\left(\dfrac{2-a}{10a^2}\right) = \left(\dfrac{3(5a)}{3(a-2)}\right)\left(\dfrac{-1(a-2)}{10a^2}\right), a \neq 0, 2$

$\left(\dfrac{15a}{3a-6}\right)\left(\dfrac{2-a}{10a^2}\right) = \left(\dfrac{5a}{a-2}\right)\left(\dfrac{-(a-2)}{10a^2}\right)$

$\left(\dfrac{15a}{3a-6}\right)\left(\dfrac{2-a}{10a^2}\right) = \dfrac{-5a(a-2)}{10a^2(a-2)}$

$\left(\dfrac{15a}{3a-6}\right)\left(\dfrac{2-a}{10a^2}\right) = \dfrac{5a(-1)(a-2)}{5a(2a)(a-2)}$

$\left(\dfrac{15a}{3a-6}\right)\left(\dfrac{2-a}{10a^2}\right) = \dfrac{-1}{2a}, a \neq 0, 2$

c) $\dfrac{m^2+2m}{m^2-4} \cdot \dfrac{12m-24}{16m}$

$= \dfrac{m(m+2)}{(m-2)(m+2)} \cdot \dfrac{4(3)(m-2)}{4(4m)}, m \neq -2, 0, 2$

$= \dfrac{m}{m-2} \cdot \dfrac{3(m-2)}{4m}$

$= \dfrac{3m(m-2)}{4m(m-2)}$

$= \dfrac{3}{4}, m \neq -2, 0, 2$

d) $\left(\dfrac{36y^2-1}{12y^2-2y}\right)\left(\dfrac{15}{30y+5}\right)$

$= \left(\dfrac{(6y+1)(6y-1)}{2y(6y-1)}\right)\left(\dfrac{5(3)}{5(6y+1)}\right), y \neq \dfrac{-1}{6}, 0, \dfrac{1}{6}$

$= \left(\dfrac{6y+1}{2y}\right)\left(\dfrac{3}{6y+1}\right)$

$= \dfrac{3(6y+1)}{2y(6y+1)}$

$= \dfrac{3}{2y}, y \neq \dfrac{-1}{6}, 0, \dfrac{1}{6}$

6. a) $\dfrac{6x^2}{15x^3} \div \dfrac{4x(x-1)}{20x^2} = \dfrac{3x^2(2)}{3x^2(5x)} \div \dfrac{4x(x-1)}{4x(5x)}, x \neq 0, 1$

$\dfrac{6x^2}{15x^3} \div \dfrac{4x(x-1)}{20x^2} = \dfrac{2}{5x} \div \dfrac{x-1}{5x}$

$\dfrac{6x^2}{15x^3} \div \dfrac{4x(x-1)}{20x^2} = \dfrac{2}{5x} \cdot \dfrac{5x}{x-1}$

$\dfrac{6x^2}{15x^3} \div \dfrac{4x(x-1)}{20x^2} = \dfrac{5x(2)}{5x(x-1)}$

$\dfrac{6x^2}{15x^3} \div \dfrac{4x(x-1)}{20x^2} = \dfrac{2}{x-1}, x \neq 0, 1$

b) $\dfrac{1-y^2}{8+8y} \div \dfrac{3y^2-3}{2y+2}$

$= \dfrac{(1-y)(1+y)}{8(1+y)} \div \dfrac{3(y-1)(y+1)}{2(y+1)}, y \neq -1, 1$

$= \dfrac{1-y}{8} \div \dfrac{3(y-1)}{2}$

$= \dfrac{-1(y-1)}{8} \cdot \dfrac{2}{3(y-1)}$

$= \dfrac{-2(y-1)}{24(y-1)}$

$= \dfrac{2(-1)(y-1)}{2(12)(y-1)}$

$= \dfrac{-1}{12}, y \neq -1, 1$

c) $\frac{9-x^2}{x^2}\div\frac{3}{x+3}=\frac{(3-x)(3+x)}{x^2}\div\frac{3}{x+3}$, $x\neq -3, 0$

$\frac{9-x^2}{x^2}\div\frac{3}{x+3}=\frac{(3-x)(x+3)}{x^2}\cdot\frac{x+3}{3}$

$\frac{9-x^2}{x^2}\div\frac{3}{x+3}=\frac{(3-x)(x+3)^2}{3x^2}$, $x\neq -3, 0$

d) $\frac{-7a}{7a-14}\div\frac{14}{a-2}=\frac{-7a}{-7(a-2)}\cdot\frac{a-2}{14}$, $a\neq 2$

$\frac{-7a}{7a-14}\div\frac{14}{a-2}=\frac{-a}{14}$, $a\neq 2$

7. a) $24x^3$ b) $6(x-2)$ c) $5(a-8)(a+8)$ d) $-2(a-3)$

8. a) $\frac{3}{x+1}-\frac{2}{3x}=\left(\frac{3}{x+1}\cdot\frac{3x}{3x}\right)-\left(\frac{2}{3x}\cdot\frac{x+1}{x+1}\right)$, $x\neq -1, 0$

$\frac{3}{x+1}-\frac{2}{3x}=\frac{9x}{3x(x+1)}-\frac{2x+2}{3x(x+1)}$

$\frac{3}{x+1}-\frac{2}{3x}=\frac{9x-2x-2}{3x(x+1)}$

$\frac{3}{x+1}-\frac{2}{3x}=\frac{7x-2}{3x(x+1)}$, $x\neq -1, 0$

b) $\frac{1}{10x}-\frac{4-x}{6x}=\left(\frac{1}{10x}\cdot\frac{3}{3}\right)-\left(\frac{4-x}{6x}\cdot\frac{5}{5}\right)$, $x\neq 0$

$\frac{1}{10x}-\frac{4-x}{6x}=\frac{3}{30x}-\frac{20-5x}{30x}$

$\frac{1}{10x}-\frac{4-x}{6x}=\frac{3-20+5x}{30x}$

$\frac{1}{10x}-\frac{4-x}{6x}=\frac{5x-17}{30x}$, $x\neq 0$

c) $3b-\frac{2}{1+3b}=\left(3b\cdot\frac{1+3b}{1+3b}\right)-\frac{2}{1+3b}$, $b\neq\frac{-1}{3}$

$3b-\frac{2}{1+3b}=\frac{3b+9b^2}{1+3b}-\frac{2}{1+3b}$

$3b-\frac{2}{1+3b}=\frac{9b^2+3b-2}{1+3b}$

$3b-\frac{2}{1+3b}=\frac{(3b+2)(3b-1)}{1+3b}$, $b\neq\frac{-1}{3}$

d) $\frac{1}{x+2}+\frac{3}{x-2}$

$=\left(\frac{1}{x+2}\cdot\frac{x-2}{x-2}\right)+\left(\frac{3}{x-2}\cdot\frac{x+2}{x+2}\right)$, $x\neq -2, 2$

$=\frac{x-2}{(x+2)(x-2)}+\frac{3x+6}{(x+2)(x-2)}$

$=\frac{x-2+3x+6}{(x+2)(x-2)}$

$=\frac{4x+4}{(x+2)(x-2)}$

$=\frac{4(x+1)}{(x+2)(x-2)}$, $x\neq -2,2$

9. a) $\frac{1}{y^2+y}+\frac{2}{1-y^2}$

$=\frac{1}{y(y+1)}+\frac{2}{(1+y)(1-y)}$, $y\neq -1, 0, 1$

$=\left(\frac{1}{y(1+y)}\cdot\frac{1-y}{1-y}\right)+\left(\frac{2}{(1+y)(1-y)}\cdot\frac{y}{y}\right)$

$=\frac{1-y}{y(1+y)(1-y)}+\frac{2y}{y(1+y)(1-y)}$

$=\frac{1-y+2y}{y(1+y)(1-y)}$

$=\frac{1+y}{y(1+y)(1-y)}$

$=\frac{1}{y(1-y)}$, $y\neq -1, 0, 1$

b) $a+\frac{a-3}{a-1}=\frac{a(a-1)}{a-1}+\frac{a-3}{a-1}$, $a\neq 1$

$a+\frac{a-3}{a-1}=\frac{a^2-3}{a-1}$, $a\neq 1$

c) $\frac{5}{2x-2}-\frac{x+4}{2x^2-2x}=\frac{5}{2(x-1)}-\frac{x+4}{2x(x-1)}$, $x\neq 0, 1$

$\frac{5}{2x-2}-\frac{x+4}{2x^2-2x}=\left(\frac{5}{2(x-1)}\cdot\frac{x}{x}\right)-\frac{x+4}{2x(x-1)}$

$\frac{5}{2x-2}-\frac{x+4}{2x^2-2x}=\frac{5x}{2x(x-1)}-\frac{x+4}{2x(x-1)}$

$\frac{5}{2x-2}-\frac{x+4}{2x^2-2x}=\frac{5x-x-4}{2x(x-1)}$

$\frac{5}{2x-2}-\frac{x+4}{2x^2-2x}=\frac{4x-4}{2x(x-1)}$

$\frac{5}{2x-2}-\frac{x+4}{2x^2-2x}=\frac{2(2)(x-1)}{2x(x-1)}$

$\frac{5}{2x-2}-\frac{x+4}{2x^2-2x}=\frac{2}{x}$, $x\neq 0, 1$

d) $\frac{5}{a-4}-\frac{a}{a^2-16}=\frac{5}{a-4}-\frac{a}{(a-4)(a+4)}$, $a\neq -4, 4$

$\frac{5}{a-4}-\frac{a}{a^2-16}=\left(\frac{5}{a-4}\cdot\frac{a+4}{a+4}\right)-\frac{a}{(a-4)(a+4)}$

$\frac{5}{a-4}-\frac{a}{a^2-16}=\frac{5a+20}{(a-4)(a+4)}-\frac{a}{(a-4)(a+4)}$

$\frac{5}{a-4}-\frac{a}{a^2-16}=\frac{5a+20-a}{(a-4)(a+4)}$

$\frac{5}{a-4}-\frac{a}{a^2-16}=\frac{4a+20}{(a-4)(a+4)}$

$\frac{5}{a-4}-\frac{a}{a^2-16}=\frac{4(a+5)}{(a-4)(a+4)}$, $a\neq -4, 4$

10. a) The non-permissible value for this equation is $x = \frac{1}{3}$.

$$\frac{5x-4}{3x-1} = \frac{3}{2},\ x \neq \frac{1}{3}$$

$$\frac{(5x-4)(2)(3x-1)}{3x-1} = \frac{3(2)(3x-1)}{2}$$

$$2(5x-4) = 3(3x-1)$$

$$10x-8 = 9x-3$$

$$10x-9x = -3+8$$

$$x = 5$$

The solution to this equation is $x = 5$.

b) The non-permissible values for this equation are $y = 10$ and $y = -10$

$$\frac{12}{10+y} + \frac{12}{10-y} = \frac{5}{2},\ x \neq 10,\ -10$$

$$\frac{12(2)(10+y)(10-y)}{10+y} + \frac{12(2)(10+y)(10-y)}{10-y} = \frac{5(2)(10+y)(10-y)}{2}$$

$$12(2)(10-y) + 12(2)(10+y) = 5(10+y)(10-y)$$

$$240 - 24y + 240 + 24y = 500 - 5y^2$$

$$480 = 500 - 5y^2$$

$$5y^2 = 500 - 480$$

$$5y^2 = 20$$

$$y^2 = 4$$

$$y = \pm 2$$

The solutions to this equation are $y = -2$ and $y = 2$.

c) The non-permissible values for this equation are $x = -4$ and $x = 16$

$$\frac{1}{2x+8} = \frac{16}{2x-32},\ x \neq -4,\ 16$$

$$\frac{1}{2(x+4)} = \frac{16}{2(x-16)}$$

$$\frac{2(x+4)(x-16)}{2(x+4)} = \frac{16(2)(x+4)(x-16)}{2(x-16)}$$

$$x-16 = 16(x+4)$$

$$x-16 = 16x+64$$

$$-15x = 80$$

$$x = \frac{-80}{15}$$

$$x = \frac{-16}{3}$$

The solution to this equation is $x = -\frac{16}{3}$.

d) The non-permissible values for this equation are $a = 3$ and $a = -3$

$$\frac{a}{3a-9} = \frac{54}{3a^2-27},\ a \neq 3,\ -3$$

$$\frac{a}{3(a-3)} = \frac{54}{3(a^2-9)}$$

$$\frac{a}{3(a-3)} = \frac{54}{3(a-3)(a+3)}$$

$$\frac{a(3)(a-3)(a+3)}{3(a-3)} = \frac{54(3)(a-3)(a+3)}{3(a-3)(a+3)}$$

$$a(a+3) = 54$$

$$a^2+3a = 54$$

$$a^2+3a-54 = 0$$

$$(a+9)(a-6) = 0$$

$$a = -9 \text{ or } a = 6$$

The solutions to this equation are $a = -9$ and $a = 6$.

11. a) Let x represent the number of comic books in the box, and let $x - 15$ represent the number of comics sold. The non-permissible values for this equation are $x = 0$ and $x = 15$. In context, x must be greater than 15, because Tayla gave 15 of the comic books she purchased to her brother.

$$1.5 = \frac{330}{x-15} - \frac{300}{x}$$

$$1.5x(x-15) = \frac{330x(x-15)}{x-15} - \frac{300x(x-15)}{x}$$

$$1.5x(x-15) = 330x - 300(x-15)$$

$$1.5x^2 - 22.5x = 330x - 300x + 4500$$

$$1.5x^2 - 22.5x = 30x + 4500$$

$$1.5x^2 - 52.5x - 4500 = 0$$

$$1.5(x^2 - 35x - 3000) = 0$$

$$1.5(x-75)(x+40) = 0$$

$$x = 75 \text{ or } x = -40$$

However, $-40 < 0$, so $x = -40$ is an inadmissible solution. Therefore, there were 75 comic books in the box.

b) $\text{Cost per comic book} = \frac{\text{Total cost}}{\text{Number of comic books}}$

$$\text{Cost per comic book} = \frac{\$300}{75}$$

$$\text{Cost per comic book} = \$4$$

Tanya paid $4 for each comic book.

12. Let x represent Dallas' speed on the highway, and let $x - 20$ represent Dallas' speed on the country roads. The non-permissible values for this equation are $x = 0$ and $x = 20$. In context, x must be greater than 20, because the highway speed is 20 km/h higher than the country road speed.

$$0.7 = \frac{20}{x} + \frac{40}{x-20}$$
$$0.7x(x-20) = \frac{20x(x-20)}{x} + \frac{40x(x-20)}{x-20}$$
$$0.7x(x-20) = 20(x-20) + 40x$$
$$0.7x^2 - 14x = 20x - 400 + 40x$$
$$0.7x^2 - 14x = 60x - 400$$
$$0.7x^2 - 14x - 60x + 400 = 0$$
$$0.7x^2 - 74x + 400 = 0$$
$$(0.7x - 4)(x - 100) = 0$$
$$x = \frac{40}{7} \quad \text{or } x = 100$$
$$x = 5.714... \text{ or } x = 100$$

However, $5.714... < 20$, so $x = 5.714...$ is an inadmissible solution. Therefore, Dallas must drive 100 km/h on the highway to complete his journey in 42 minutes.

Chapter Task, page 267

A. When the rational expressions are in factored form, it is easier to identify common factors for division.

B. The 12 rational expressions need to share some common factors because dividing by factors is the only way to score points. Sometimes, after adding or subtracting, the resulting expression may have common factors, but this will not happen very often.

C., D. To maximize your score, always make sure that you try to find common factors at the beginning and at the end. When "wild ops" is rolled, choosing division or multiplication will more likely lead to scoring points. If you analyze the other dice carefully and quickly, you may be able to identify which choice will score the most points.

Chapter 4 Diagnostic Test, TR page 249

1. a) $\frac{3}{4} + \frac{7}{4} = \frac{3+7}{4}$

$\frac{3}{4} + \frac{7}{4} = \frac{10}{4}$

$\frac{3}{4} + \frac{7}{4} = \frac{5}{2}$

b) $\frac{2}{3} - \frac{5}{6} = \frac{2}{3} \cdot \frac{2}{2} - \frac{5}{6}$

$\frac{2}{3} - \frac{5}{6} = \frac{4}{6} - \frac{5}{6}$

$\frac{2}{3} - \frac{5}{6} = -\frac{1}{6}$

c) $\frac{5}{6} \cdot \frac{2}{10} = \frac{5 \cdot 2}{6 \cdot 10}$

$\frac{5}{6} \cdot \frac{2}{10} = \frac{10}{60}$

$\frac{5}{6} \cdot \frac{2}{10} = \frac{1}{6}$

d) $\frac{6}{15} \div \frac{3}{5} = \frac{6}{15} \cdot \frac{5}{3}$

$\frac{6}{15} \div \frac{3}{5} = \frac{6 \cdot 5}{15 \cdot 3}$

$\frac{6}{15} \div \frac{3}{5} = \frac{30}{45}$

$\frac{6}{15} \div \frac{3}{5} = \frac{2}{3}$

e) $\frac{\frac{2}{5}}{\frac{4}{3}} = \frac{2}{5} \cdot \frac{3}{4}$

$\frac{\frac{2}{5}}{\frac{4}{3}} = \frac{2 \cdot 3}{5 \cdot 4}$

$\frac{\frac{2}{5}}{\frac{4}{3}} = \frac{6}{20}$

$\frac{\frac{2}{5}}{\frac{4}{3}} = \frac{3}{10}$

f) $\left(\frac{3}{5}\right)^2 \cdot \left(-\frac{10}{6}\right) = \left(\frac{3}{5}\right) \cdot \left(\frac{3}{5}\right) \cdot \left(-\frac{10}{6}\right)$

$\left(\frac{3}{5}\right)^2 \cdot \left(-\frac{10}{6}\right) = \frac{3 \cdot 3 \cdot (-10)}{5 \cdot 5 \cdot 6}$

$\left(\frac{3}{5}\right)^2 \cdot \left(-\frac{10}{6}\right) = -\frac{90}{150}$

$\left(\frac{3}{5}\right)^2 \cdot \left(-\frac{10}{6}\right) = -\frac{3}{5}$

2. a) $8x^3 - 10x = 2x(4x^2 - 5)$
b) $y^2 - 81 = (y - 9)(y + 9)$
c) $4xy^2 - 16x = 4x(y^2 - 4)$
$4xy^2 - 16x = 4x(y - 2)(y + 2)$
d) $x^2 - 7x + 12 = x^2 - 4x - 3x + 12$
$x^2 - 7x + 12 = (x - 4)(x - 3)$

3. a) $4x^2 - 8x = 0$
Factoring:
$4x(x - 2) = 0$
$4x = 0$ or $x - 2 = 0$
$x = 0$ or $x = 2$

b) $4x^2 - 9 = 0$
Factoring:

$$4\left(x^2 - \frac{9}{4}\right) = 0$$

$$4\left(x - \frac{3}{2}\right)\left(x + \frac{3}{2}\right) = 0$$

$$x - \frac{3}{2} = 0 \text{ or } x + \frac{3}{2} = 0$$

$$x = \frac{3}{2} \text{ or } \quad x = -\frac{3}{2}$$

c) $x^2 - 5x - 6 = 0$
Factoring:
$(x - 6)(x + 1) = 0$
$x - 6 = 0$ or $x + 1 = 0$
$x = 6$ or $\quad x = -1$
d) $3x^3 = 75x$
Factoring:
$3x^3 - 75x = 0$
$3x(x^2 - 25) = 0$
$3x(x - 5)(x + 5) = 0$
$3x = 0$ or $x - 5 = 0$ or $x + 5 = 0$
$x = 0$ or $\quad x = 5$ or $\quad x = -5$

4. Let x represent the number of hours Joanne drove on the highway, and let y represent the number of hours she drove on rural roads.
She drove on the highway at an average speed of 90 km/h, and on rural roads at an average speed of 45 km/h. Her total distance was 340 km. This can be modelled by the equation:
(90 km/h)x + (45 km/h)y = 340 km
or $90x + 45y = 340$ (1)
This makes sense because x and y are measured in hours.

The trip took 4 h, which can be modelled by the equation:
$x + y = 4$ h (2)

Since there are two equations with two unknown variables, I can solve for a variable. Since I am looking for the distance Joanne drove on the highway I will solve for x.

Putting y in terms of x using equation (2):
$y = 4 - x$ (3)

Substituting equation (3) into equation (1):
$90x + 45(4 - x) = 340$
Solving for x:

$$\begin{aligned} 90x + 180 - 45x &= 340 \\ 45x + 180 &= 340 \\ x &= 160 \\ x &= 3.555\ldots \text{ h} \end{aligned}$$

Calculating the distance:
90 km/h · 3.555...h = 319.999... km
Joanne travelled about 320 km along the highway.

5. Let t represent the time it will take to fill the pool when both hoses are used.

$$\frac{1}{12} + \frac{1}{8} = \frac{1}{t}$$

LCD: $24t$

$$\frac{1}{12} \cdot 24t + \frac{1}{8} \cdot 24t = \frac{1}{t} \cdot 24t$$

$$\begin{aligned} 2t + 3t &= 24 \\ 5t &= 24 \\ t &= \frac{24}{5} \end{aligned}$$

It will take $\frac{24}{5}$ or 4.8 h to fill the pool if both hoses are used.

Review of Terms and Connections, TR page 251

1. a) i) rational number: any number that can be written in the form, $\frac{m}{n}$, where m and n are integers and n does not equal zero
b) iv) equivalent rational numbers: numbers that have the same location on the number line
c) v) common factors: an integer that divides into two or more other integers evenly
d) vi) lowest common denominator: a number that is needed to add or subtract rational numbers
e) viii) binomial: $3x^2 + 6x$
f) ii) algebraic expression: a mathematical statement that contains numbers, variables, and/or operations; for example, $\frac{b \cdot h}{2}$
g) vii) factored expression: $3x(x + 2)$
h) iii) quotient: the result of a division
i) ix) numerator: the top of a fraction

2. a) $\frac{3}{5} + \frac{1}{5} = \frac{3+1}{5}$

$$\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$$

b) $\frac{3}{5} + \frac{1}{3} = \frac{3}{5} \cdot \frac{3}{3} + \frac{1}{3} \cdot \frac{5}{5}$

$$\frac{3}{5} + \frac{1}{3} = \frac{9}{15} + \frac{5}{15}$$

$$\frac{3}{5} + \frac{1}{3} = \frac{9+5}{15}$$

$$\frac{3}{5} + \frac{1}{3} = \frac{14}{15}$$

c) $\frac{5}{3}-\frac{1}{6}=\frac{5}{3}\cdot\frac{2}{2}-\frac{1}{6}$

$\frac{5}{3}-\frac{1}{6}=\frac{10}{6}-\frac{1}{6}$

$\frac{5}{3}-\frac{1}{6}=\frac{10-1}{6}$

$\frac{5}{3}-\frac{1}{6}=\frac{9}{6}$

$\frac{5}{3}-\frac{1}{6}=\frac{3}{2}$

d) $\frac{7}{4}-\frac{5}{9}=\frac{7}{4}\cdot\frac{9}{9}-\frac{5}{9}\cdot\frac{4}{4}$

$\frac{7}{4}-\frac{5}{9}=\frac{63}{36}-\frac{20}{36}$

$\frac{7}{4}-\frac{5}{9}=\frac{63-20}{36}$

$\frac{7}{4}-\frac{5}{9}=\frac{43}{36}$

3. a) $\frac{3}{5}\cdot\frac{5}{2}=\frac{3\cdot5}{5\cdot2}$

$\frac{3}{5}\cdot\frac{5}{2}=\frac{15}{10}$

$\frac{3}{5}\cdot\frac{5}{2}=\frac{3}{2}$

b) $\frac{9}{4}\cdot\frac{2}{3}=\frac{9\cdot2}{4\cdot3}$

$\frac{9}{4}\cdot\frac{2}{3}=\frac{18}{12}$

$\frac{9}{4}\cdot\frac{2}{3}=\frac{3}{2}$

c) $\frac{5}{3}\cdot6=\frac{5\cdot6}{3}$

$\frac{5}{3}\cdot6=\frac{30}{3}$

$\frac{5}{3}\cdot6=10$

d) $\frac{7}{4}\cdot\frac{10}{5}=\frac{7\cdot10}{4\cdot5}$

$\frac{7}{4}\cdot\frac{10}{5}=\frac{70}{20}$

$\frac{7}{4}\cdot\frac{10}{5}=\frac{7}{2}$

4. a) $\frac{7}{5}\div\frac{14}{3}=\frac{7}{5}\cdot\frac{3}{14}$

$\frac{7}{5}\div\frac{14}{3}=\frac{21}{70}$

$\frac{7}{5}\div\frac{14}{3}=\frac{3}{10}$

b) $\frac{5}{9}\div\frac{11}{9}=\frac{5}{9}\cdot\frac{9}{11}$

$\frac{5}{9}\div\frac{11}{9}=\frac{45}{99}$

$\frac{5}{9}\div\frac{11}{9}=\frac{5}{11}$

c) $\frac{4}{3}\div6=\frac{4}{3}\cdot\frac{1}{6}$

$\frac{4}{3}\div6=\frac{4}{18}$

$\frac{4}{3}\div6=\frac{2}{9}$

d) $\frac{\frac{7}{4}}{\frac{3}{8}}=\frac{7}{4}\cdot\frac{8}{3}$

$\frac{\frac{7}{4}}{\frac{3}{8}}=\frac{56}{12}$

$\frac{\frac{7}{4}}{\frac{3}{8}}=\frac{14}{3}$

5. a) $x^2-9x=0$

Factoring: $x(x-9)=0$

$x=0$ or $x-9=0$

$x=0$ or $x=9$

b) $x^2+2x=-1$

$x^2+2x+1=0$

Factoring: $(x+1)^2=0$

$x+1=0$

$x=-1$

c) $4x^2-1=0$

$x^2-\frac{1}{4}=0$

Factoring: $\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)=0$

$x-\frac{1}{2}=0$ or $x+\frac{1}{2}=0$

$x=\frac{1}{2}$ or $x=-\frac{1}{2}$

6. a) $x^2 - 9x = 2x^2 + 4x + 36$
Set quadratic equation to zero:
$0 = 2x^2 - x^2 + 4x + 9x + 36$
$x^2 + 13x + 36 = 0$
Factoring: $(x + 9)(x + 4) = 0$
$x + 9 = 0$ or $x + 4 = 0$
$x = -9$ or $x = -4$

b) $\frac{x}{6} + \frac{x}{2} = \frac{3x}{2} + 5$ Multiply by 6 to get rid of fractions:
$x + 3x = 9x + 30$
Solving for x:
$9x - x - 3x = -30$
$5x = -30$
$x = -6$

c) $x(x - 1) = -x + 16$
Setting quadratic equation to zero:
$x^2 - x = -x + 16$
$x^2 - 16 = 0$
Factoring: $(x - 4)(x + 4) = 0$
$x - 4 = 0$ or $x + 4 = 0$
$x = 4$ or $x = -4$

d) $\frac{3x}{2} = \frac{5}{6}$ Multiply by 6 to get rid of fractions:
$9x = 5$ Solving for x:
$x = \frac{5}{9}$

7. area = length · width

length = depth = $\frac{1}{3}$ mile

width = frontage = $\frac{3}{4} \div \frac{1}{2}$

width = $\frac{3}{4} \div \frac{1}{2} \cdot \frac{2}{2}$

width = $\frac{3}{4} \div \frac{2}{4}$

width = $\frac{5}{4}$ or $1\frac{1}{4}$ mile

area = $\frac{1}{3} \cdot \frac{5}{4}$

area = $\frac{5}{12}$

The total area of the properties is $\frac{5}{12}$ square miles.

8. e.g., Suppose that you wanted to find out how many $\frac{1}{2}$-acre parcels of land fit into 7 acres. You would need to divide by a fraction: $7 \div \frac{1}{2}$.

9. Let t represent the time, in days, it takes for Dale and Tania to paint the house together. The fraction of the house painted in one day by Dale is $\frac{1}{3}$, and by Tania is $\frac{1}{2}$.

$\frac{1}{3} + \frac{1}{2} = \frac{1}{t}$
$\frac{2}{6} + \frac{3}{6} = \frac{1}{t}$
$\frac{5}{6} = \frac{1}{t}$
$\frac{5t}{6} = \frac{t}{t}$
$\frac{5t}{6} = 1$
$5t = 6$
$t = \frac{6}{5}$

It would take Dale and Tania $\frac{6}{5}$ days to paint the house if they work together.

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1. a) $\frac{15x^5}{35x^7} = \frac{3}{7x^2}, x \neq 0$

b) $\frac{2x^3 - 18x}{4x - 12} = \frac{2x(x^2 - 9)}{4(x - 3)}, x \neq 3$
$\frac{2x^3 - 18x}{4x - 12} = \frac{x(x^2 - 9)}{2(x - 3)}, x \neq 3$
$\frac{2x^3 - 18x}{4x - 12} = \frac{2x(x - 3)(x + 3)}{4(x - 3)}, x \neq 3$
$\frac{2x^3 - 18x}{4x - 12} = \frac{x(x + 3)}{2}, x \neq 3$

2. a) $\frac{3x^2}{x+1} \cdot \frac{x+1}{9} = \frac{3x^2(x+1)}{9(x+1)}, x \neq -1$
$\frac{3x^2}{x+1} \cdot \frac{x+1}{9} = \frac{x^2}{3}, x \neq -1$

b) $\frac{4a^2 - 1}{4a} \div \frac{2a+1}{2a} = \frac{4a^2 - 1}{4a} \cdot \frac{2a}{2a+1}, x \neq -\frac{1}{2}, 0$
$\frac{4a^2 - 1}{4a} \div \frac{2a+1}{2a} = \frac{(4a^2 - 1)(2a)}{4a(2a+1)}, x \neq -\frac{1}{2}, 0$
$\frac{4a^2 - 1}{4a} \div \frac{2a+1}{2a} = \frac{(2a-1)(2a+1)(2a)}{4a(2a+1)}, x \neq -\frac{1}{2}, 0$
$\frac{4a^2 - 1}{4a} \div \frac{2a+1}{2a} = \frac{2a-1}{2}, x \neq -\frac{1}{2}, 0$

3. a) $\frac{x^2-10}{5x}+\frac{4x^2}{5x}=\frac{(x^2-10)+4x^2}{5x},\ x\neq 0$

$\frac{x^2-10}{5x}+\frac{4x^2}{5x}=\frac{5x^2-10}{5x},\ x\neq 0$

$\frac{x^2-10}{5x}+\frac{4x^2}{5x}=\frac{5(x^2-2)}{5x},\ x\neq 0$

$\frac{x^2-10}{5x}+\frac{4x^2}{5x}=\frac{x^2-2}{x},\ x\neq 0$

b) $\frac{3}{2x-6}-\frac{1}{2x}$

$=\left(\frac{3}{2x-6}\right)\cdot\left(\frac{2x}{2x}\right)-\left(\frac{1}{2x}\right)\cdot\left(\frac{3}{2x-6}\right),\ x\neq 0,\ 3$

$=\frac{6x}{4x^2-12x}-\frac{2x-6}{4x^2-12x},\ x\neq 0,\ 3$

$=\frac{6x-(2x-6)}{4x^2-12x},\ x\neq 0,\ 3$

$=\frac{4x-6}{4x^2-12x},\ x\neq 0,\ 3$

$=\frac{2(2x-3)}{4x(x-3)},\ x\neq 0,\ 3$

$=\frac{2x-3}{2x(x-3)},\ x\neq 0,\ 3$

4. a) Let x represent Leanne's speed on flat terrain:

$\text{time}=\frac{\text{distance}}{\text{speed}}$

$\frac{24x}{(x-3)(x+3)},\ x\neq -3,\ 3$

b) $\frac{34x^2-90}{x(x-3)(x+3)},\ x\neq -3,\ 0,\ 3$

c) The non-permissible values are $x = -3, 0, 3$; these values would result in division by zero. The inadmissible values are $x \leq 3$; the hills slow down Leanne by 3 km/h, so she must be travelling faster than 3 km/h.

5. a) $\frac{1}{4x}+\frac{3}{8x}=-5$

$2+3=-5(8x)$

$5=-40x$

$-\frac{5}{40}=x$

$-\frac{1}{8}=x$

b) $\frac{4}{a+2}+\frac{2}{a-2}=\frac{6a}{a^2-4}$

$\frac{4}{a+2}+\frac{2}{a-2}=\frac{6a}{(a-2)(a+2)}$

$\left(\frac{4}{a+2}+\frac{2}{a-2}\right)\cdot(a-2)(a+2)=\left(\frac{6a}{(a-2)(a+2)}\right)(a-2)(a+2)$

$4(a-2)+2(a+2)=6a$

$4a-8+2a+4=6a$

$6a-4=6a$

$-4=0$

There is no solution.

6. Nathan's first error occurred in the step after he multiplied by the LCD. He did not simplify the second term correctly. Nathan also made an error when he set the equation equal to zero; "+18" should be "–18." The correct solution is given below.

$\frac{2a+6}{a^2}+\frac{1}{3a}=\frac{1}{3},\ a\neq 0$

$\text{LCD}=3a^2$

$\left(\frac{2a+6}{a^2}\right)\cdot(3a^2)+\left(\frac{1}{3a}\right)\cdot(3a^2)=\left(\frac{1}{3}\right)\cdot(3a^2)$

$3(2a+6)+a=a^2$

$6a+18+a=a^2$

$0=a^2-7a-18$

$0=(a-9)(a+2)$

$a = -2$ or $a = 9$

7. Let x represent Damien's average speed on a bike. Then model Damien's average speed in a car by $x + 40$. The time Damien took to drive to the shop is

$\frac{30\text{ km}}{x\text{ km/h}}=\frac{30\text{ h}}{x}$

The time Damien took to ride home is

$\frac{30\text{ km}}{x+40\text{ km/h}}=\frac{30\text{ h}}{x+40}$

The times for each leg of the trip should sum to 2 h.

$\frac{30\text{ h}}{x}+\frac{30\text{ h}}{x+40}=2\text{ h}$

Solve for x to determine Damien's average speed.

$\frac{30}{x}+\frac{30}{x+40}=2$

$30(x+40)+30(x)=2(x)(x+40)$

$30x+1200+30x=2x^2+80x$

$0=2x^2+20x-1200$

$0=2(x+30)(x-20)$

$x = -30$ or $x = 20$

Since a speed can't be negative, –30 is not a solution. Damien's average speed on his bike was 20 km/h.

8. Let x represent the time it takes Sandra to deliver the papers alone, in minutes.

$$\frac{1}{x}+\frac{1}{x+30}=\frac{1}{36}$$

$$\left(\frac{1}{x}+\frac{1}{x+30}\right)(36)(x)(x+30)=\left(\frac{1}{36}\right)(36)(x)(x+30)$$

$$36(x+30)+36x=(x)(x+30)$$

$$36x+1080+36x=x^2+30x$$

$$0=x^2-42x-1080$$

$$0=(x-60)(x+18)$$

$x = 60$ or $x = -18$

Since time cannot be negative, $x = 60$.

It takes Sandra 60 min to deliver the paper alone, and it takes Naomi 30 min more. Naomi takes 90 min to deliver the papers alone.